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FEMIX 4.0

FORMULAS

Note: a vector is stored in a column matrix

Example: $\underline{x} = (x_1, x_2, x_3) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [x_1 \quad x_2 \quad x_3]^T$

Strain operator

$$\underline{\varepsilon} = \underline{L}\ \underline{u}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 & 0 \\ 0 & \frac{\partial}{\partial x_2} & 0 \\ 0 & 0 & \frac{\partial}{\partial x_3} \\ 0 & \frac{\partial}{\partial x_3} & \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_3} & 0 & \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\underline{L} = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 & 0 \\ 0 & \frac{\partial}{\partial x_2} & 0 \\ 0 & 0 & \frac{\partial}{\partial x_3} \\ 0 & \frac{\partial}{\partial x_3} & \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_3} & 0 & \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} & 0 \end{bmatrix}$$

Stress strain relation (isotropic materials)

$$\underline{\sigma} = \underline{D}\ \underline{\varepsilon}$$

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{bmatrix} = \begin{bmatrix} C_1 & C_2 & C_2 & 0 & 0 & 0\\ C_2 & C_1 & C_2 & 0 & 0 & 0\\ C_2 & C_2 & C_1 & 0 & 0 & 0\\ 0 & 0 & 0 & C_3 & 0 & 0\\ 0 & 0 & 0 & 0 & C_3 & 0\\ 0 & 0 & 0 & 0 & 0 & C_3\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{bmatrix}$$

$$C_1 = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$C_2 = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$C_3 = \frac{E}{2(1+\nu)}$$

$$\underline{D} = \begin{bmatrix} C_1 & C_2 & C_2 & 0 & 0 & 0\\ C_2 & C_1 & C_2 & 0 & 0 & 0\\ C_2 & C_2 & C_1 & 0 & 0 & 0\\ 0 & 0 & 0 & C_3 & 0 & 0\\ 0 & 0 & 0 & 0 & C_3 & 0\\ 0 & 0 & 0 & 0 & 0 & C_3\end{bmatrix}$$

Coordinate transformation

$$\begin{cases} x_1 = x_1(s_1, s_2, s_3)\\ x_2 = x_2(s_1, s_2, s_3)\\ x_3 = x_3(s_1, s_2, s_3)\end{cases}$$

Jacobian matrix $(m \times m)$

$$\underline{J} = \begin{bmatrix} \frac{\partial x_1}{\partial s_1} & \frac{\partial x_1}{\partial s_2} & \frac{\partial x_1}{\partial s_3}\\ \frac{\partial x_2}{\partial s_1} & \frac{\partial x_2}{\partial s_2} & \frac{\partial x_2}{\partial s_3}\\ \frac{\partial x_3}{\partial s_1} & \frac{\partial x_3}{\partial s_2} & \frac{\partial x_3}{\partial s_3}\end{bmatrix}$$

Determinant of the Jacobian matrix

$$J = \left| \underline{J} \right|$$

Interpolation of the thickness of a laminate element

Note: $\bar{h}_i$ is the thickness of the element in node i

$$\underline{\bar{h}} = \begin{bmatrix} \bar{h}_1 \\ \bar{h}_2 \\ \vdots \\ \bar{h}_n \end{bmatrix}$$

$$h = \bar{h}_1 N_1 + \bar{h}_2 N_2 + \cdots + \bar{h}_n N_n$$

$$h = \begin{bmatrix} \bar{h}_1 & \bar{h}_2 & \cdots & \bar{h}_n \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_n \end{bmatrix}$$

$$h = \underline{\bar{h}}^T \; \underline{N}_V$$

Interpolation of the cartesian coordinates of the element nodes

$$\begin{cases} x_1 = \bar{x}_{11} N_1 + \bar{x}_{21} N_2 + \cdots + \bar{x}_{n1} N_n \\ x_2 = \bar{x}_{12} N_1 + \bar{x}_{22} N_2 + \cdots + \bar{x}_{n2} N_n \\ x_3 = \bar{x}_{13} N_1 + \bar{x}_{23} N_2 + \cdots + \bar{x}_{n3} N_n \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \bar{x}_{11} & \bar{x}_{21} & \cdots & \bar{x}_{n1} \\ \bar{x}_{12} & \bar{x}_{22} & \cdots & \bar{x}_{n2} \\ \bar{x}_{13} & \bar{x}_{23} & \cdots & \bar{x}_{n3} \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_n \end{bmatrix}$$

$$\underline{x} = \underline{\bar{x}}^T \; \underline{N}_V$$

Interpolation of the displacements of the element nodes

$$\begin{cases} u_1 = N_1 a_{11} + N_2 a_{21} + \cdots + N_n a_{n1} \\ u_2 = N_1 a_{12} + N_2 a_{22} + \cdots + N_n a_{n2} \\ u_3 = N_1 a_{13} + N_2 a_{23} + \cdots + N_n a_{n3} \end{cases}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \left[\begin{array}{ccc|ccc|c|ccc} N_1 & 0 & 0 & N_2 & 0 & 0 & & N_n & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \cdots & 0 & N_n & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & & 0 & 0 & N_n \end{array}\right] \left[\begin{array}{c} a_{11} \\ a_{12} \\ a_{13} \\ \hline a_{21} \\ a_{22} \\ a_{23} \\ \hline \vdots \\ \hline a_{n1} \\ a_{n2} \\ a_{n3} \end{array}\right]$$

$$\underline{u} = \underline{N}\ \underline{a}$$

$$\underline{N} = \left[\begin{array}{ccc|ccc|c|ccc} N_1 & 0 & 0 & N_2 & 0 & 0 & & N_n & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \cdots & 0 & N_n & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & & 0 & 0 & N_n \end{array}\right]$$