Lajes pelo MEF

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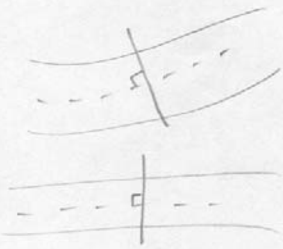
2005-12-13

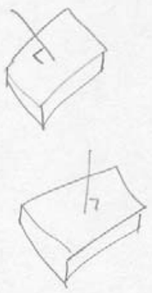
LAJ-1

http://www.fe.up.pt/~alvaro

Viga de Euler-Bernoulli

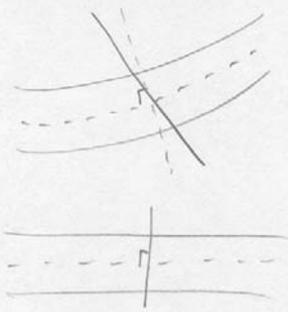


Laje de Kirchhoff

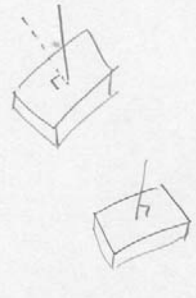


Não se consideram deformações devidas a tensões tangenciais
(lajes delgadas)

Viga de Timoshenko

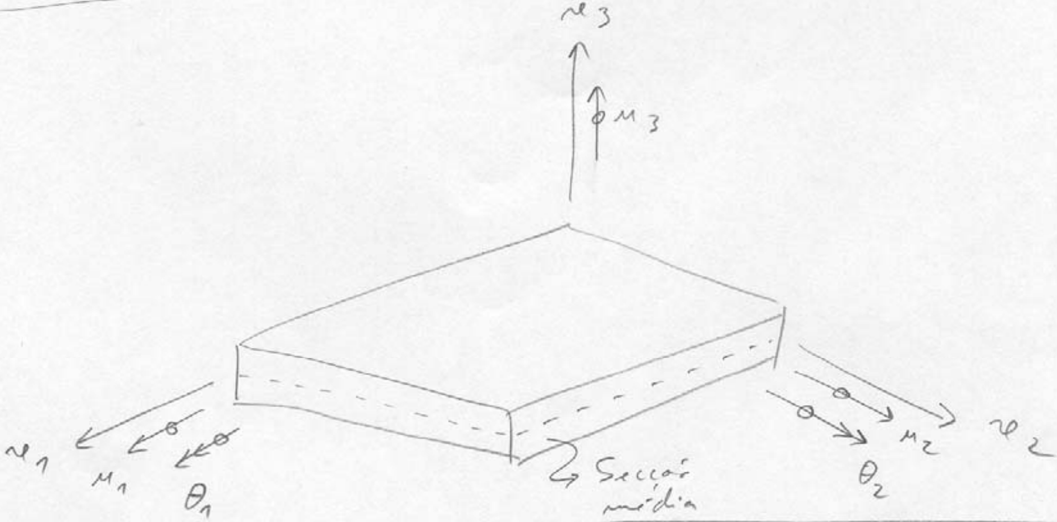


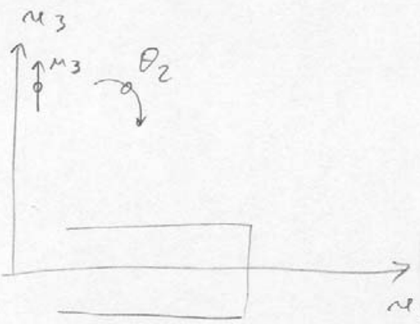
Laje de Reissner - Mindlin

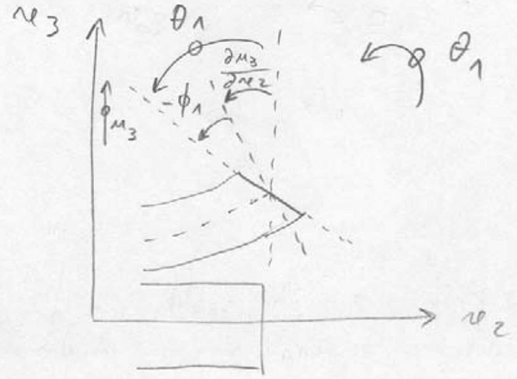


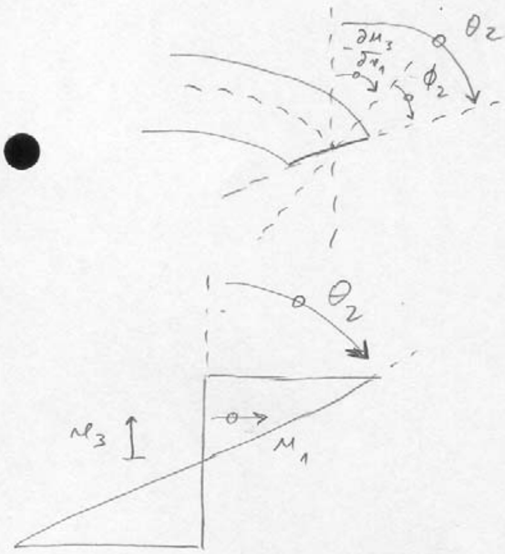
São consideradas as deformações devidas a tensões tangenciais
(lajes espessas)

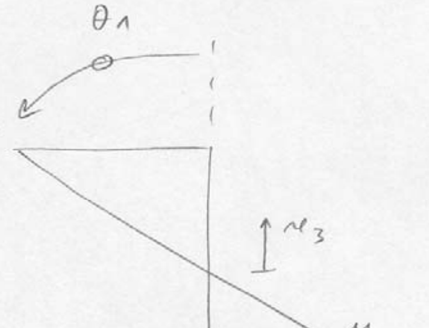
Laje de Reissner - Mindlin











$$\theta_2 = -\frac{\partial u_3}{\partial x_1} + \phi_2 \qquad \theta_1 = \frac{\partial u_3}{\partial x_2} - \phi_1$$

$$\phi_2 = \frac{\partial u_3}{\partial x_1} + \theta_2 \qquad \phi_1 = \frac{\partial u_3}{\partial x_2} - \theta_1$$

Campo de deslocamentos

$$\begin{cases} u_1 = x_3 \ \theta_2 \\ u_2 = -x_3 \ \theta_1 \end{cases}$$

$$\begin{cases} u_1(x_1, x_2, x_3) = x_3 \ \theta_2(x_1, x_2) \\ u_2(x_1, x_2, x_3) = -x_3 \ \theta_1(x_1, x_2) \end{cases}$$

$$u_3(x_1, x_2, x_3) = u_3(x_1, x_2)$$ → deslocamento da secção média segundo x_3

Campo de deformações

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial u_1}{\partial x_1} \\ \dfrac{\partial u_2}{\partial x_2} \\ \dfrac{\partial u_3}{\partial x_3} \\ \dfrac{\partial u_2}{\partial x_3} + \dfrac{\partial u_3}{\partial x_2} \\ \dfrac{\partial u_3}{\partial x_1} + \dfrac{\partial u_1}{\partial x_3} \\ \dfrac{\partial u_1}{\partial x_2} + \dfrac{\partial u_2}{\partial x_1} \end{bmatrix} = \begin{bmatrix} x_3 \dfrac{\partial \theta_2}{\partial x_1} \\ -x_3 \dfrac{\partial \theta_1}{\partial x_2} \\ 0 \\ -\theta_1 + \dfrac{\partial u_3}{\partial x_2} \\ \dfrac{\partial u_3}{\partial x_1} + \theta_2 \\ x_3 \dfrac{\partial \theta_2}{\partial x_2} - x_3 \dfrac{\partial \theta_1}{\partial x_1} \end{bmatrix} = \begin{bmatrix} x_3 \dfrac{\partial \theta_2}{\partial x_1} \\ -x_3 \dfrac{\partial \theta_1}{\partial x_2} \\ 0 \\ \phi_1 \\ \phi_2 \\ x_3 \left(\dfrac{\partial \theta_2}{\partial x_2} - \dfrac{\partial \theta_1}{\partial x_1} \right) \end{bmatrix}$$

Reagrupando e uma vez que $\gamma_{13} = \gamma_{31}$, tem-se

$$\underline{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \hdashline \gamma_{23} \\ \gamma_{13} \end{bmatrix} = \begin{bmatrix} x_3 \left(\frac{\partial \theta_2}{\partial x_1} \right) \\ x_3 \left(- \frac{\partial \theta_1}{\partial x_2} \right) \\ x_3 \left(\frac{\partial \theta_2}{\partial x_2} - \frac{\partial \theta_1}{\partial x_1} \right) \\ \hdashline \frac{\partial u_3}{\partial x_2} - \theta_1 \\ \frac{\partial u_3}{\partial x_1} + \theta_2 \end{bmatrix} = \begin{bmatrix} \underline{\varepsilon}_b \\ \hdashline \underline{\varepsilon}_s \end{bmatrix}$$

$$\underline{\varepsilon}_b = x_3 \begin{bmatrix} \frac{\partial \theta_2}{\partial x_1} \\ -\frac{\partial \theta_1}{\partial x_2} \\ \frac{\partial \theta_2}{\partial x_2} - \frac{\partial \theta_1}{\partial x_1} \end{bmatrix} = x_3 \, \underline{\bar{\varepsilon}}_b$$

$$\underline{\bar{\varepsilon}}_b = \begin{bmatrix} \frac{\partial \theta_2}{\partial x_1} \\ -\frac{\partial \theta_1}{\partial x_2} \\ \frac{\partial \theta_2}{\partial x_2} - \frac{\partial \theta_1}{\partial x_1} \end{bmatrix}$$

$$\underline{\varepsilon}_s = \begin{bmatrix} \frac{\partial u_3}{\partial x_2} - \theta_1 \\ \frac{\partial u_3}{\partial x_1} + \theta_2 \end{bmatrix}$$

$$\underline{\varepsilon} = \begin{bmatrix} x_3 \; \underline{\varepsilon}_b \\ \hline \underline{\varepsilon}_s \end{bmatrix}$$

Campo de tensões

$$\underline{\sigma} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \hline \tau_{23} \\ \tau_{13} \end{bmatrix} = \begin{bmatrix} \underline{\sigma}_b \\ \hline \underline{\sigma}_s \end{bmatrix}$$

$$\underline{\sigma}_b = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \quad ; \quad \underline{\sigma}_s = \begin{bmatrix} \tau_{23} \\ \tau_{13} \end{bmatrix}$$

Relações entre tensões e deformações

Estado plano de tensão:

$$\underbrace{\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}}_{\underline{\sigma}_b} = \underbrace{\begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix}}_{\underline{D}_b} \underbrace{\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}}_{\underline{\varepsilon}_b}$$

$$\underline{\sigma}_b = \underline{D}_b \; \underline{\varepsilon}_b$$

$$\begin{bmatrix}\tau_{23}\\ \tau_{13}\end{bmatrix} = \begin{bmatrix}\alpha G & 0\\ 0 & \alpha G\end{bmatrix}\begin{bmatrix}\gamma_{23}\\ \gamma_{13}\end{bmatrix}$$

$\alpha = \frac{5}{6}$ (fator correctivo de corte em secções rectangulares)

$$\underline{\sigma}_s = \underline{D}_s \; \underline{\varepsilon}_s$$

$$\underline{D}_s = \begin{bmatrix}\alpha G & 0\\ 0 & \alpha G\end{bmatrix}$$

$$\left[\begin{array}{c}\underline{\sigma}_b\\ \hline \underline{\sigma}_s\end{array}\right] = \left[\begin{array}{c|c}\underline{D}_b & \underline{0}\\ \hline \underline{0} & \underline{D}_s\end{array}\right]\left[\begin{array}{c}\underline{\varepsilon}_b\\ \underline{\varepsilon}_s\end{array}\right]$$

$$\underline{\sigma} = \underline{D} \; \underline{\varepsilon}$$

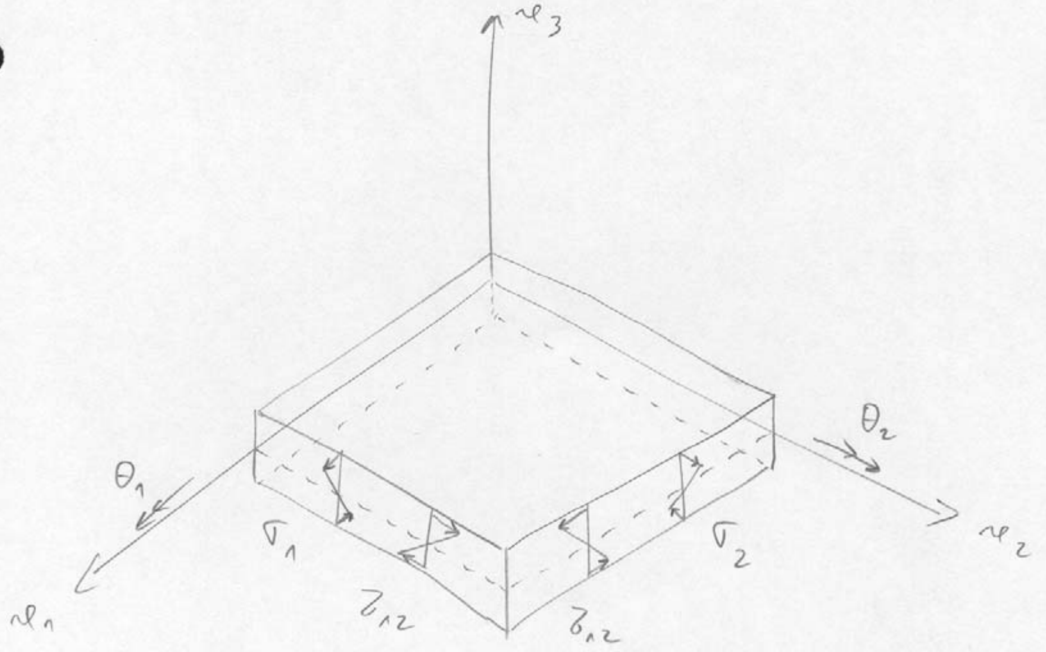
LAJ - 7

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_1 & C_2 & 0 \\ C_2 & C_1 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

$$\begin{cases} \sigma_1 = C_1 \varepsilon_1 + C_2 \varepsilon_2 \\ \sigma_2 = C_2 \varepsilon_1 + C_1 \varepsilon_2 \\ \tau_{12} = C_3 \gamma_{12} \end{cases}$$

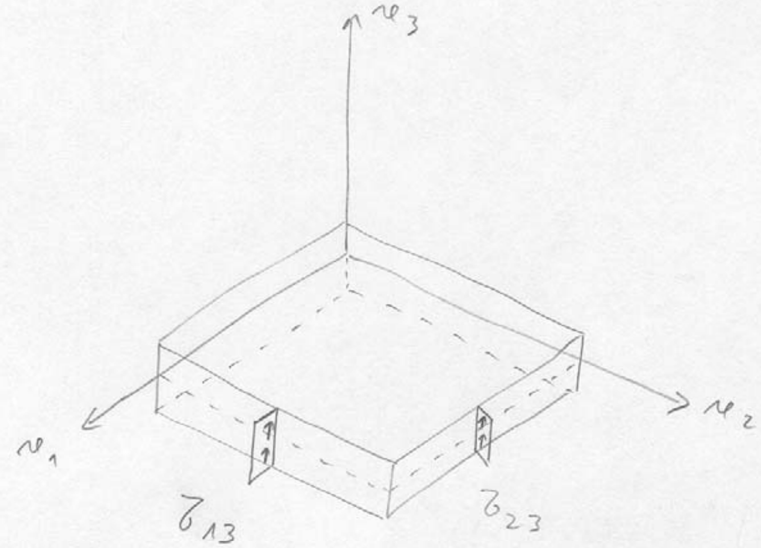
$$\begin{cases} \sigma_1 = C_1 x_3 \dfrac{\partial \theta_2}{\partial x_1} - C_2 x_3 \dfrac{\partial \theta_1}{\partial x_2} = \left(C_1 \dfrac{\partial \theta_2}{\partial x_1} - C_2 \dfrac{\partial \theta_1}{\partial x_2} \right) x_3 \\ \sigma_2 = C_2 x_3 \dfrac{\partial \theta_2}{\partial x_1} - C_1 x_3 \dfrac{\partial \theta_1}{\partial x_2} = \left(C_2 \dfrac{\partial \theta_2}{\partial x_1} - C_1 \dfrac{\partial \theta_1}{\partial x_2} \right) x_3 \\ \tau_{12} = C_3 x_3 \left(\dfrac{\partial \theta_2}{\partial x_2} - \dfrac{\partial \theta_1}{\partial x_1} \right) = C_3 \left(\dfrac{\partial \theta_2}{\partial x_2} - \dfrac{\partial \theta_1}{\partial x_1} \right) x_3 \end{cases}$$

σ_1, σ_2 e τ_{12} são funções lineares de x_3



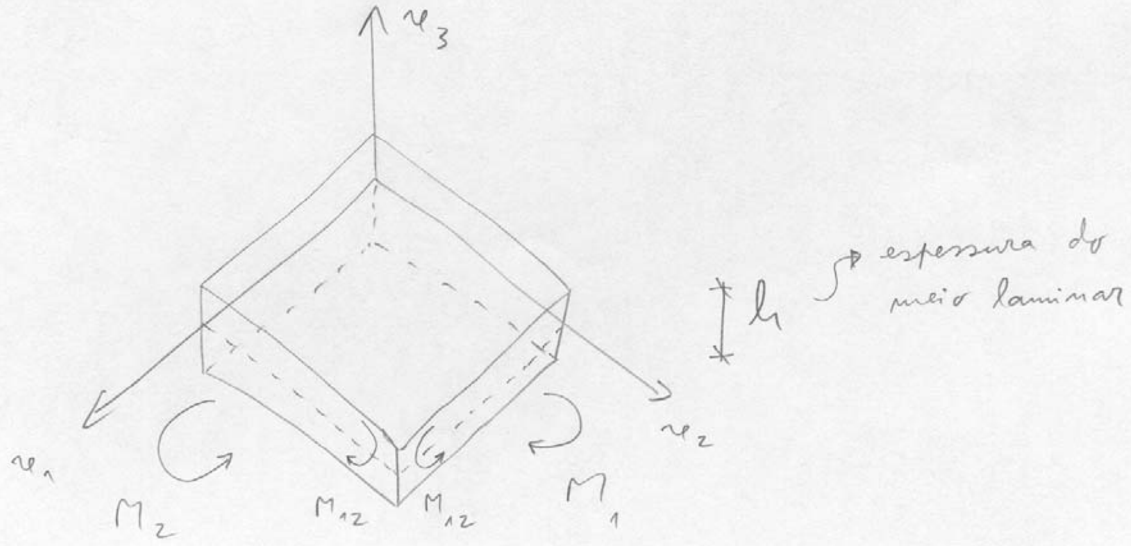
$$\begin{cases} \tau_{23} = \alpha\, G\, \gamma_{23} = \alpha\, G \left(\dfrac{\partial u_3}{\partial x_2} - \theta_1 \right) \\[2ex] \tau_{13} = \alpha\, G\, \gamma_{13} = \alpha\, G \left(\dfrac{\partial u_3}{\partial x_1} + \theta_2 \right) \end{cases}$$

τ_{23} e τ_{13} são constantes ao longo da espessura



Esforços

Momentos por unidade de comprimento



LAJ - 9

$$M_2 = \int_{-h/2}^{h/2} \sigma_1 \, x_3 \, dx_3$$

$$M_1 = \int_{-h/2}^{h/2} \sigma_2 \, x_3 \, dx_3$$

$$M_{12} = \int_{-h/2}^{h/2} \tau_{12} \, x_3 \, dx_3$$

- $$\underline{M} = \begin{bmatrix} M_2 \\ M_1 \\ M_{12} \end{bmatrix} = \int_{-h/2}^{h/2} x_3 \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} d x_3$$

$$\underline{M} = \int_{-h/2}^{h/2} x_3 \, \underline{\sigma}_b \, dx_3$$

- $$\underline{M} = \int_{-h/2}^{h/2} x_3 \, \underline{D}_b \, \underline{\varepsilon}_b \, dx_3$$

$$\underline{M} = \int_{-h/2}^{h/2} x_3 \, \underline{D}_b \, x_3 \, \underline{\bar{\varepsilon}}_b \, dx_3$$

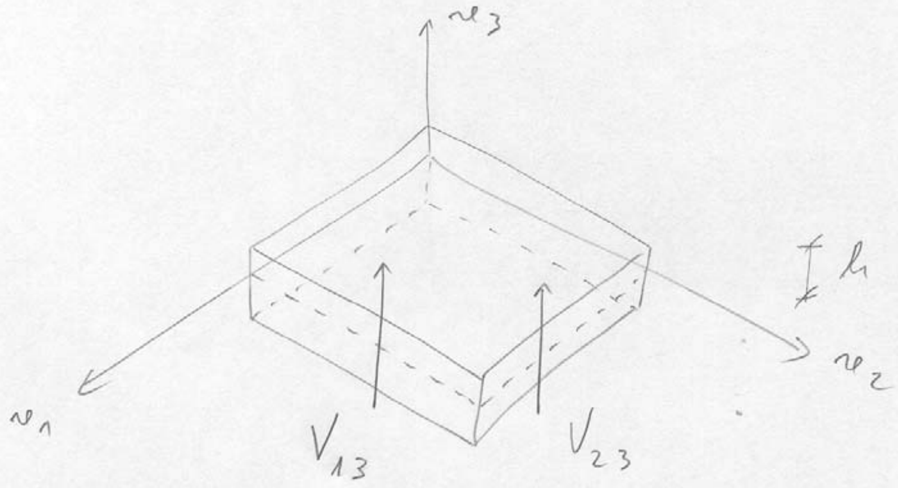
$$\underline{M} = \left(\int_{-h/2}^{h/2} x_3^2 \, dx_3 \right) \underline{D}_b \, \bar{\underline{\varepsilon}}_b$$

$$\underline{M} = \frac{h^3}{12} \underline{D}_b \, \bar{\underline{\varepsilon}}_b$$

$$\underline{M} = \bar{\underline{D}}_b \, \bar{\underline{\varepsilon}}_b$$

$$\bar{\underline{D}}_b = \frac{h^3}{12} \underline{D}_b$$

- Esforço transverso por unidade de comprimento



$$V_{23} = \int_{-h/2}^{h/2} \tau_{23} \, dx_3$$

$$V_{13} = \int_{-h/2}^{h/2} \tau_{13} \, dx_3$$

$$\underline{V} = \begin{bmatrix} V_{23} \\ V_{13} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \tau_{23} \\ \tau_{13} \end{bmatrix} dx_3$$

$$\underline{V} = \int_{-h/2}^{h/2} \underline{\tau}_s \; dx_3$$

$$\underline{V} = \int_{-h/2}^{h/2} \underline{D}_s \; \underline{\varepsilon}_s \; dx_3$$

- $$\underline{V} = \left(\int_{-h/2}^{h/2} dx_3 \right) \underline{D}_s \; \underline{\varepsilon}_s$$

$$\underline{V} = h \; \underline{D}_s \; \underline{\varepsilon}_s$$

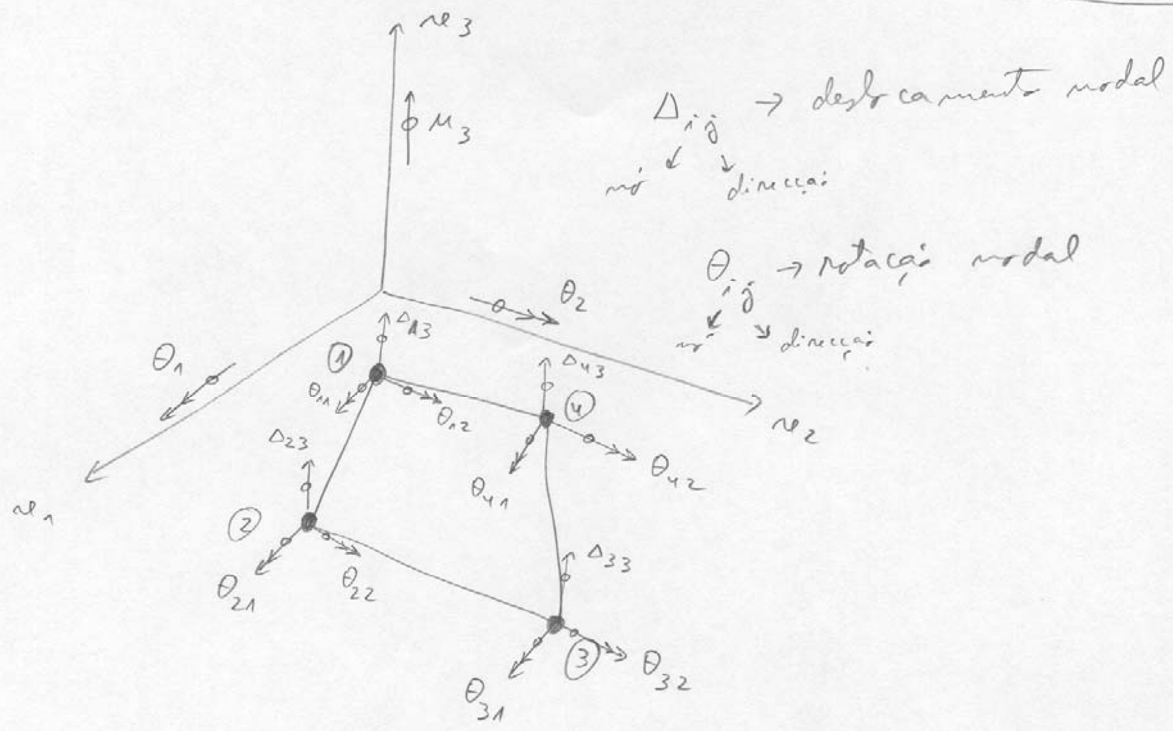
$$\underline{V} = \underline{\bar{D}}_s \; \underline{\varepsilon}_s$$

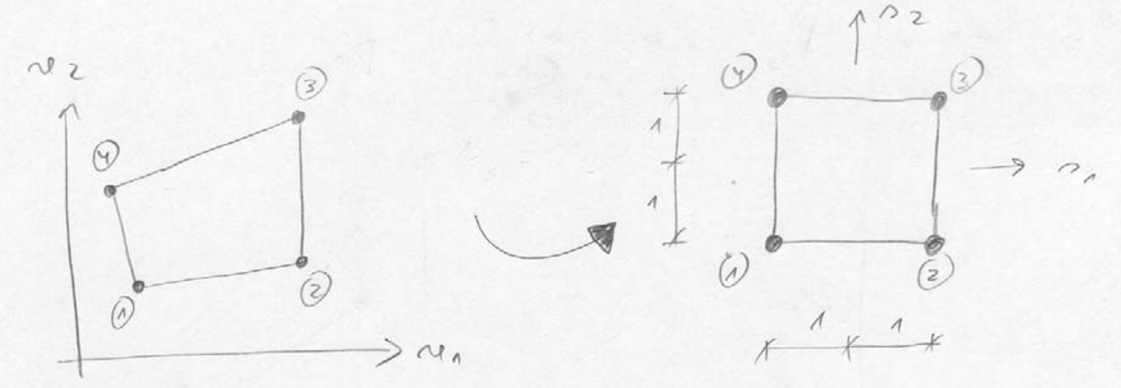
- $$\underline{\bar{D}}_s = h \; \underline{D}_s$$

$$\begin{bmatrix} \underline{M} \\ \hline \underline{V} \end{bmatrix} = \left[\begin{array}{c|c} \underline{\bar{D}}_b & \underline{0} \\ \hline \underline{0} & \underline{\bar{D}}_s \end{array}\right] \begin{bmatrix} \underline{\bar{\varepsilon}}_b \\ \hline \underline{\varepsilon}_s \end{bmatrix}$$

$$\underline{\bar{\sigma}} = \underline{\bar{D}} \; \underline{\bar{\varepsilon}}$$

Elemento finito de 4 nós





Tal como no estado plano de tensão:

$$\underline{x} = \underline{\bar{x}}^T \underline{N}_v$$

$$h = \underline{\bar{h}}^T N_v$$

$$\underline{J} = \underline{\bar{x}}^T \frac{\partial \underline{N}}{\partial \underline{s}} \qquad J = |\underline{J}|$$

$$\frac{\partial \underline{N}}{\partial \underline{x}} = \frac{\partial \underline{N}}{\partial \underline{s}} \underline{J}^{-1}$$

LAJ-13

Interpolação separada dos campos de deslocamentos e de rotações

$$\begin{cases} u_3(x_1,x_2) = N_1(x_1,x_2)\,\Delta_{13} + N_2(x_1,x_2)\,\Delta_{23} + N_3(x_1,x_2)\,\Delta_{33} + N_4(x_1,x_2)\,\Delta_{43} \\ \theta_1(x_1,x_2) = N_1(x_1,x_2)\,\theta_{11} + N_2(x_1,x_2)\,\theta_{21} + N_3(x_1,x_2)\,\theta_{31} + N_4(x_1,x_2)\,\theta_{41} \\ \theta_2(x_1,x_2) = N_1(x_1,x_2)\,\theta_{12} + N_2(x_1,x_2)\,\theta_{22} + N_3(x_1,x_2)\,\theta_{32} + N_4(x_1,x_2)\,\theta_{42} \end{cases}$$

$$\underline{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \hdashline a_4 \\ a_5 \\ a_6 \\ \hdashline a_7 \\ a_8 \\ a_9 \\ \hdashline a_{10} \\ a_{11} \\ a_{12} \end{bmatrix} = \begin{bmatrix} \Delta_{13} \\ \theta_{11} \\ \theta_{12} \\ \hdashline \Delta_{23} \\ \theta_{21} \\ \theta_{22} \\ \hdashline \Delta_{33} \\ \theta_{31} \\ \theta_{32} \\ \hdashline \Delta_{43} \\ \theta_{41} \\ \theta_{42} \end{bmatrix}$$

$$\underline{\overline{\varepsilon}}_b = \begin{bmatrix} \dfrac{\partial \theta_2}{\partial x_1} \\ -\dfrac{\partial \theta_1}{\partial x_2} \\ \dfrac{\partial \theta_2}{\partial x_2} - \dfrac{\partial \theta_1}{\partial x_1} \end{bmatrix}$$

LAJ-14

$$\underline{\bar{\varepsilon}}_b = \left[\begin{array}{ccc|c|ccc} 0 & 0 & \frac{\partial N_1}{\partial x_1} & & 0 & 0 & \frac{\partial N_4}{\partial x_1} \\ 0 & -\frac{\partial N_1}{\partial x_2} & 0 & \cdots & 0 & -\frac{\partial N_4}{\partial x_2} & 0 \\ 0 & -\frac{\partial N_1}{\partial x_1} & \frac{\partial N_1}{\partial x_2} & & 0 & -\frac{\partial N_4}{\partial x_1} & \frac{\partial N_4}{\partial x_2} \end{array}\right] \begin{bmatrix} \Delta_{13} \\ \theta_{11} \\ \theta_{12} \\ \hline \vdots \\ \hline \Delta_{43} \\ \theta_{41} \\ \theta_{42} \end{bmatrix}$$

- $$\underset{(3\times 1)}{\underline{\bar{\varepsilon}}_b} = \underset{(3\times 12)}{\underline{B}_b}\ \underset{(12\times 1)}{\underline{a}}$$

$$\underline{\varepsilon}_s = \begin{bmatrix} \frac{\partial u_3}{\partial x_2} - \theta_1 \\ \frac{\partial u_3}{\partial x_1} + \theta_2 \end{bmatrix}$$

- $$\underline{\varepsilon}_s = \left[\begin{array}{ccc|c|ccc} \frac{\partial N_1}{\partial x_2} & -N_1 & 0 & & \frac{\partial N_4}{\partial x_2} & -N_4 & 0 \\ & & & \cdots & & & \\ \frac{\partial N_1}{\partial x_1} & 0 & N_1 & & \frac{\partial N_4}{\partial x_1} & 0 & N_4 \end{array}\right] \begin{bmatrix} \Delta_{13} \\ \theta_{11} \\ \theta_{12} \\ \hline \vdots \\ \hline \Delta_{43} \\ \theta_{41} \\ \theta_{42} \end{bmatrix}$$

$$\underset{(2\times 1)}{\underline{\varepsilon}_s} = \underset{(2\times 12)}{\underline{B}_s}\ \underset{(12\times 1)}{\underline{a}}$$

Princípio dos trabalhos virtuais

Trabalho Interno = Trabalho Externo

$$\delta W_b^i + \delta W_s^i = \delta W^e$$

$$\delta W_b^i = \int_V \delta \underline{\varepsilon}_b^T \, \underline{\sigma}_b \, dV$$

$dV = dx_3 \, dS$ sendo $dS = dx_1 \, dx_2$

$$\delta W_b^i = \int_S \int_{-h/2}^{h/2} \delta \underline{\varepsilon}_b^T \, \underline{\sigma}_b \, dx_3 \, dS$$

$$\underline{\varepsilon}_b = x_3 \, \underline{\bar{\varepsilon}}_b$$

$$\underline{\bar{\varepsilon}}_b = \underline{B}_b \, \underline{a}$$

$$\underline{\varepsilon}_b = x_3 \, \underline{B}_b \, \underline{a}$$

$$\delta \underline{\varepsilon}_b^T = \delta \underline{a}^T \, \underline{B}_b^T \, x_3$$

$$\underline{\sigma}_b = \underline{D}_b \, \underline{\varepsilon}_b$$

$$\underline{\sigma}_b = \underline{D}_b \, x_3 \, \underline{\bar{\varepsilon}}_b$$

$$\underline{\sigma}_b = \underline{D}_b \, x_3 \, \underline{B}_b \, \underline{a}$$

$$\delta W_b^i = \int_S \int_{-h/2}^{h/2} \delta \underline{a}^T \, \underline{B}_b^T \, x_3 \, \underline{D}_b \, x_3 \, \underline{B}_b \, \underline{a} \; dx_3 \, dS$$

$$\delta W_b^i = \delta \underline{a}^T \int_S \underline{B}_b^T \, \underline{D}_b \, \underline{B}_b \int_{-h/2}^{h/2} x_3^2 \, dx_3 \; dS \; \underline{a}$$

$$\delta W_b^i = \delta \underline{a}^T \int_S \underline{B}_b^T \, \frac{h^3}{12} \, \underline{D}_b \, \underline{B}_b \; dS \; \underline{a}$$

$$\delta W_b^i = \delta \underline{a}^T \int_S \underline{B}_b^T \, \underline{\bar{D}}_b \, \underline{B}_b \; dS \; \underline{a}$$

$$\delta W_s^i = \int_V \delta \underline{\varepsilon}_s^T \, \underline{\sigma}_s \; dV$$

$$\delta W_s^i = \int_S \int_{-h/2}^{h/2} \delta \underline{\varepsilon}_s^T \, \underline{\sigma}_s \; dx_3 \, dS$$

$$\underline{\varepsilon}_s = \underline{B}_s \, \underline{a}$$

$$\delta \underline{\varepsilon}_s^T = \delta \underline{a}^T \, \underline{B}_s^T$$

$$\underline{\sigma}_s = \underline{D}_s \, \underline{\varepsilon}_s$$

$$\underline{\sigma}_s = \underline{D}_s \, \underline{B}_s \, \underline{a}$$

$$\delta W_s^i = \int_S \int_{-h/2}^{h/2} \delta \underline{a}^T \underline{B}_s^T \underline{D}_s \underline{B}_s \underline{a} \, dx_3 \, dS$$

$$\delta W_s^i = \delta \underline{a}^T \int_S \underline{B}_s^T \underline{D}_s \underline{B}_s \int_{-h/2}^{h/2} dx_3 \, dS \, \underline{a}$$

$$\delta W_s^i = \delta \underline{a}^T \int_S \underline{B}_s^T h \underline{D}_s \underline{B}_s \, dS \, \underline{a}$$

$$\delta W_s^i = \delta \underline{a}^T \int_S \underline{B}_s^T \underline{\bar{D}}_s \underline{B}_s \, dS \, \underline{a}$$

$$\delta W_b^i + \delta W_s^i = \delta W^e$$

$$\delta \underline{a}^T \int_S \underline{B}_b^T \underline{\bar{D}}_b \underline{B}_b \, dS \, \underline{a} + \delta \underline{a}^T \int_S \underline{B}_s^T \underline{\bar{D}}_s \underline{B}_s \, dS \, \underline{a} = \delta \underline{a}^T \underline{F}$$

$$\underline{K} \, \underline{a} = \underline{F}$$

$$K = \underline{K}_b + \underline{K}_s$$

$$\underline{K}_b = \int_S \underline{B}_b^T \underline{\bar{D}}_b \underline{B}_b \, dS$$

$$\underline{K}_s = \int_S \underline{B}_s^T \underline{\bar{D}}_s \underline{B}_s \, dS$$

LAJ-18

$$\int_S [\cdots]\, dS = \int_{-1}^{+1}\int_{-1}^{+1} [\cdots]\; J\, dr_1\, dr_2$$

$$\int_{-1}^{+1}\int_{-1}^{+1} f(r_1, r_2)\, dr_1\, dr_2 =$$

$$= \sum_i \sum_j W_i W_j f(P_i, P_j)$$

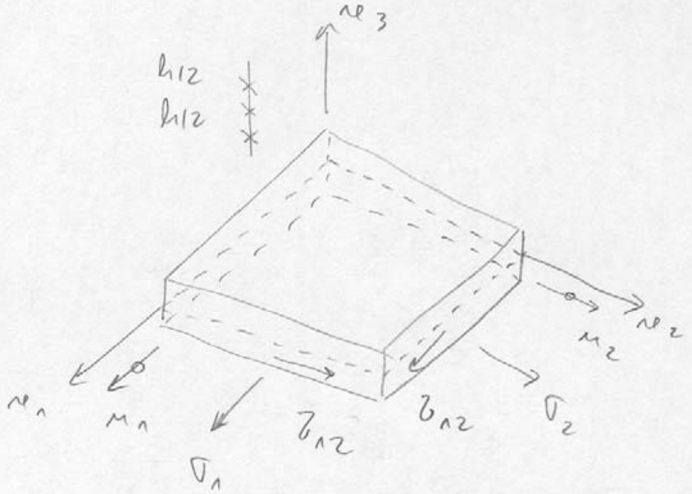
CP - 1

Cascas planas (Reissner - Mindlin)

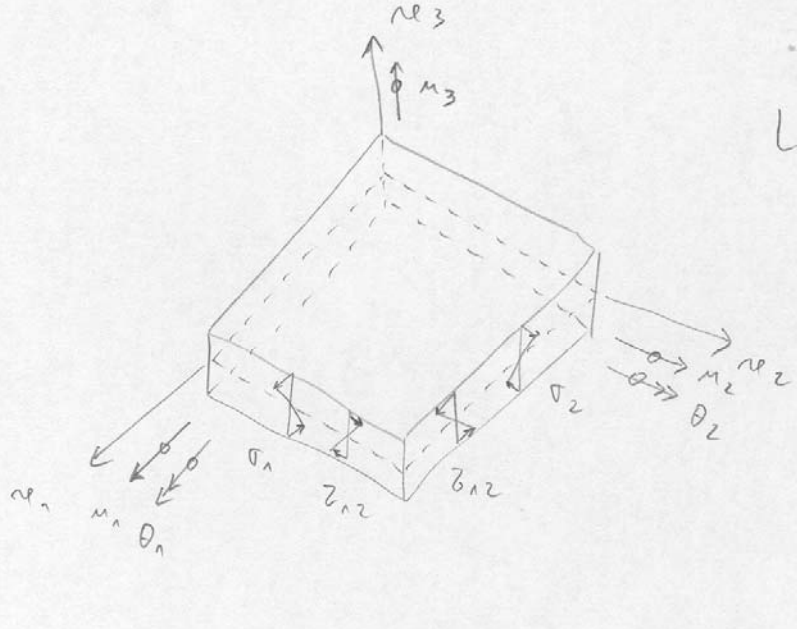
Álvaro Azevedo 2006-01-03

Considera-se simultaneamente o comportamento como

- membrana
- laje de Reissner - Mindlin

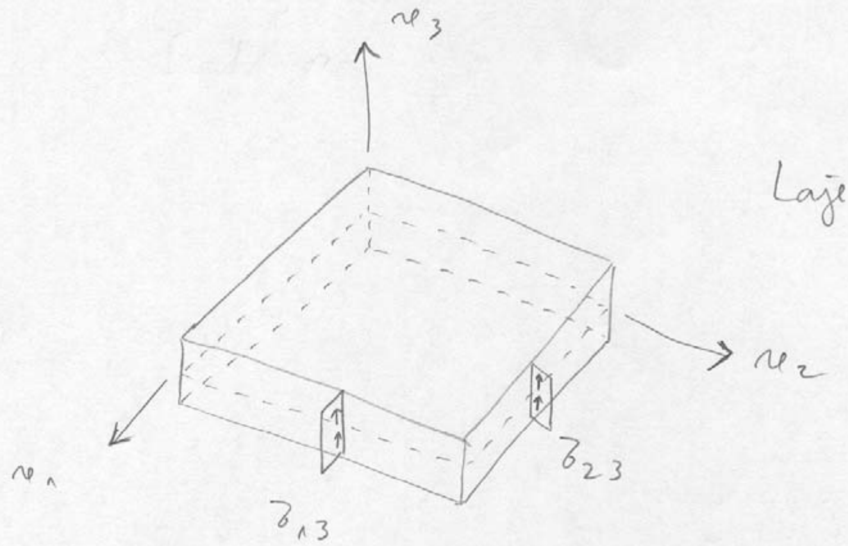


Membrana



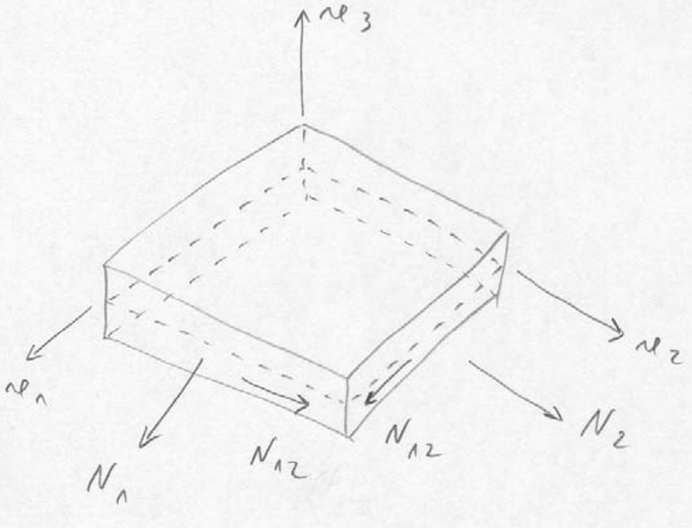
Laje - flexão

CP-2

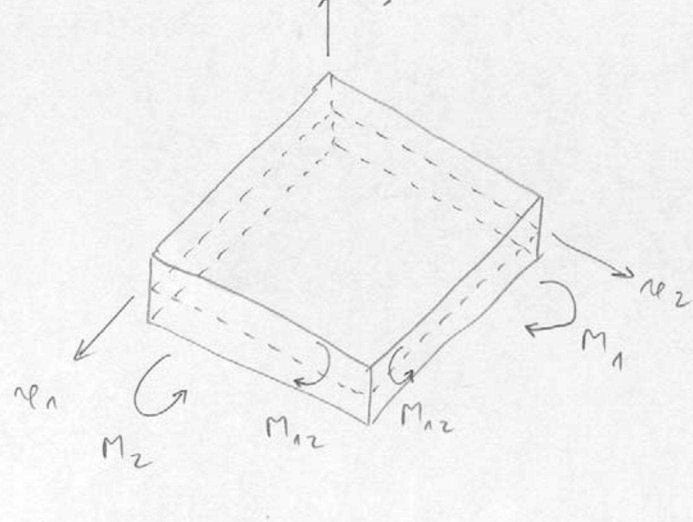


Laje - corte

Esforços (por unidade de comprimento)

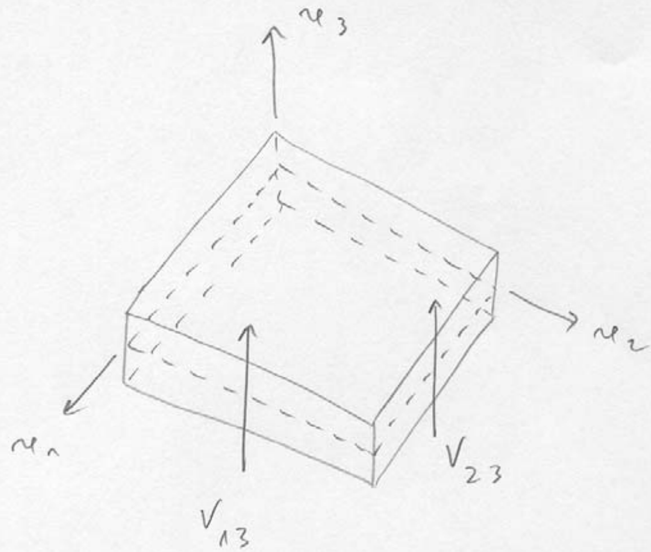


Membrana

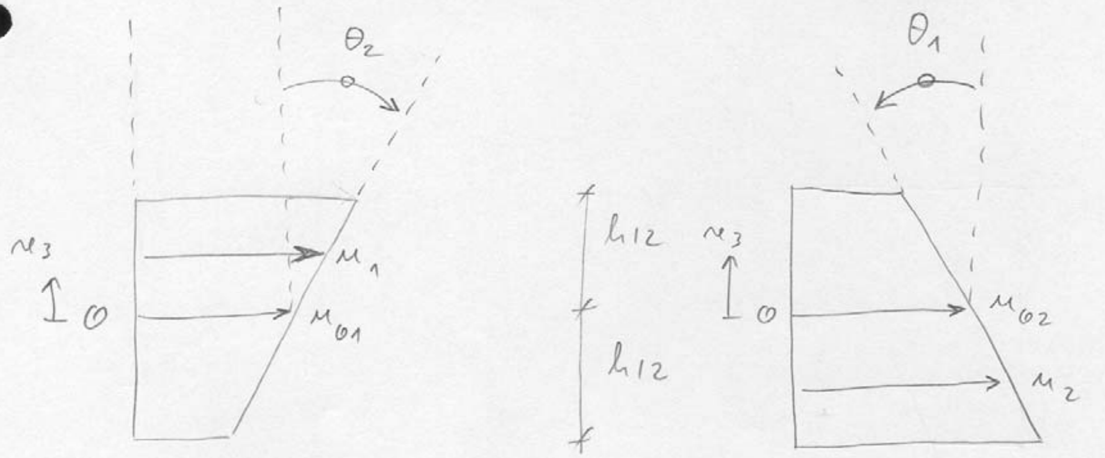


Laje - flexão

CP-3



Laje - corte

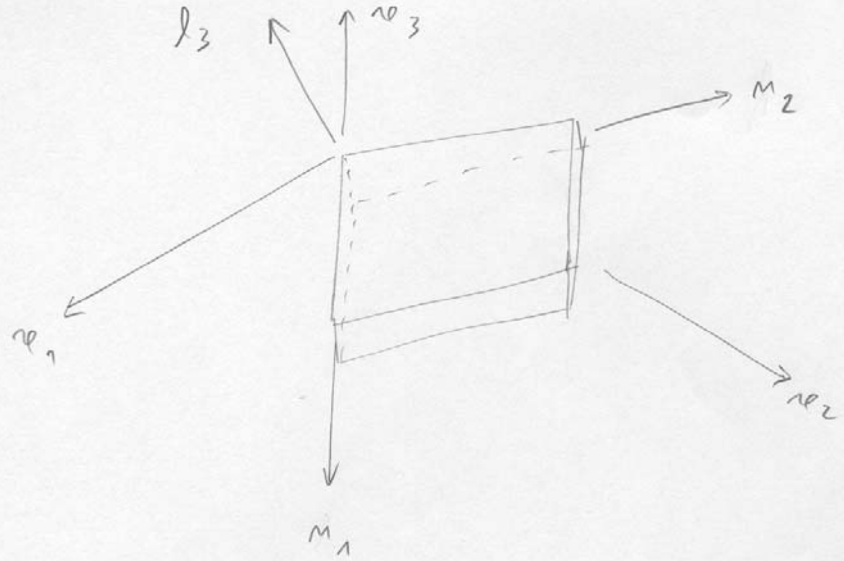


$$\theta_2 = -\frac{\partial u_3}{\partial x_1} + \phi_2 \qquad \theta_1 = \frac{\partial u_3}{\partial x_2} - \phi_1$$

$$\begin{cases} u_1 = u_{01} + x_3 \ \theta_2 \\ u_2 = u_{02} - x_3 \ \theta_1 \end{cases}$$

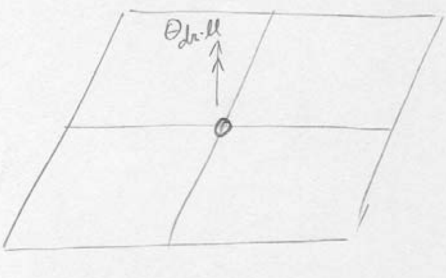
$$\begin{cases} u_1(x_1, x_2, x_3) = u_{01}(x_1, x_2) + x_3 \ \theta_2(x_1, x_2) \\ u_2(x_1, x_2, x_3) = u_{02}(x_1, x_2) - x_3 \ \theta_1(x_1, x_2) \end{cases}$$

$$u_3(x_1, x_2, x_3) = u_{03}(x_1, x_2)$$

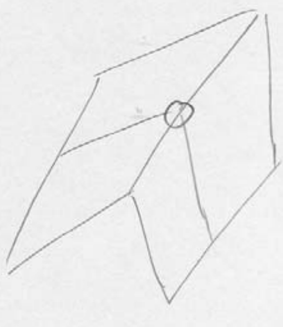


- Referencial nodal n → anteriormente designado x
  Referencial geral x

$$\underline{n} = \underline{T}\ \underline{x}$$

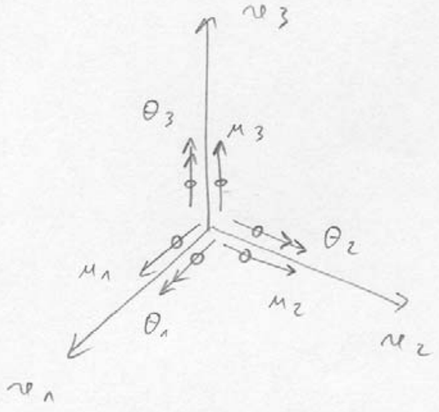
- 

Nó coplanar
(sem drill dof)



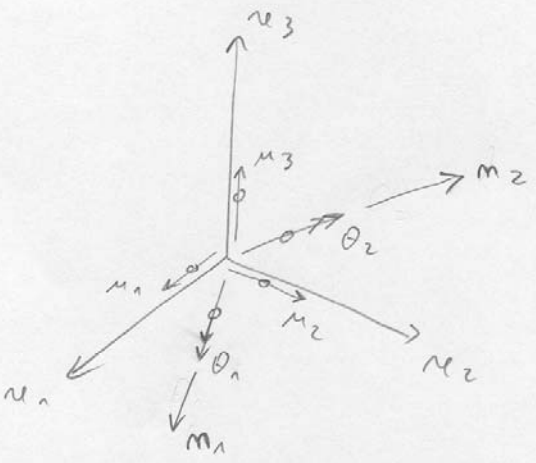
Nó não coplanar
Nó de aresta
Nó de kink

Nó de Kink:



u_1, u_2, u_3
$\theta_1, \theta_2, \theta_3$
} referencial geral x

Nó coplanar:

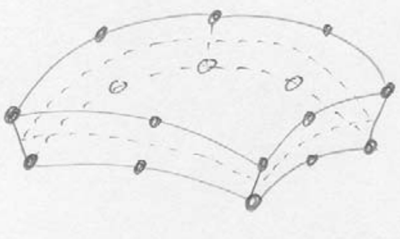


$u_1, u_2, u_3 \rightarrow$ referencial geral x

$\theta_1, \theta_2 \rightarrow$ referencial nodal m

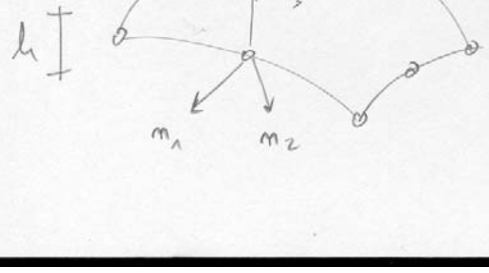
Elemento de casca de Ahmad

Sólido degenerado



Superfície média
com dupla curvatura

Referencial nodal varia de nó para nó

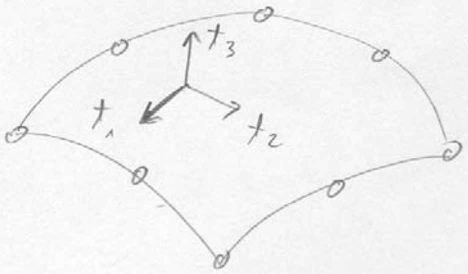


$\rightarrow$ Elemento finito
representativo da
superfície média

$h \rightarrow$ variável

Referencial local t

CP-6



$$\begin{cases} t_1 , t_2 \rightarrow \text{tangentes à superfície média} \\ t_3 \rightarrow \text{normal à superfície média} \end{cases}$$

Links:

http://civil.fe.up.pt/pub/apoio/ano5/aae/ano5_aae.htm

http://civil.fe.up.pt/pub/apoio/ano5/aae/pdf/Apontamentos/Manuscritos/Lajes_e_Cascas_pelo_MEF.pdf