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# A 3D Time Domain Formulation for the Analysis of Train Induced Vibrations

ISMA2010 - International Conference on Noise and Vibration Engineering, Leuven, Belgium

pp. 3441-3451, September 2010

# A 3D time domain formulation for the analysis of train induced vibrations

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## Abstract

This paper describes the implementation and validation of a numerical methodology which can be used in the prediction of vehicle induced vibrations. The presented methodology is formulated in the time domain for the three-dimensional case. Although presented in the context of vehicle induced vibrations it can be adopted in the study of general problems of dynamic soil-structure interaction in which the structure lies on the surface of the soil. The method is based on the coupling of the fundamental Green functions to a finite element code (FEMIX). Some relevant aspects of the implementation are briefly discussed and a numerical example based on the study of the dynamic response of a slab resting on a homogeneous halfspace and subjected to the passage of a moving load is presented.

## 1 Introduction

A considerable effort has been undertaken in recent years to develop numerical methods for the study of problems involving a dynamic soil-structure interaction (SSI). The simulation of problems involving train induced vibrations requires an adequate consideration of the dynamic track-soil interaction. The expansion of the high-speed railway lines contributed to the actual relevance of this issue.

Due to its versatility, the Finite Element Method (FEM) is a very popular and attractive tool. However, this method presents some limitations when used in the simulation of this type of problems. Due to its semi-infinite nature, the soil cannot be fully discretized and, consequently, some type of domain truncation is required. These artificial boundaries lead to spurious reflections, which affect the behaviour of the modelled part of the system. If the discretized domain is very large (in order to move the artificial boundaries away from the region of the model with interest), the problem becomes computationally intractable. Problems related to spurious reflections may be minimized (but not fully eliminated) using absorbing boundaries (viscous dampers connected to the boundaries of the finite element model) [1], Perfectly Matched Layers (PML) [2] or infinite elements [3].

As an alternative, some authors applied hybrid methods combining the Finite Element Method (FEM) and the Boundary Element Method (BEM) in order to take advantage of the versatility of FEM and the capability of the BEM to simulate infinite media [4].

Other authors developed numerical models based on a 2.5D concept, e.g., 2.5D finite/infinite elements (FEM/IEM) [5] and 2.5D FEM/BEM [6]. These procedures are very efficient from the computational point of view. The main drawback of these models is associated with the assumption of invariability of the geometry and properties of the structure along one direction. Moreover they are usually limited to linear or equivalent linear analyses.

The present paper describes the implementation and application of a three-dimensional numerical methodology formulated in the time domain and based on the coupling of the fundamental Green Functions (GF) to a FE code [7, 8]. These functions reproduce the behaviour of the soil while the FE model enables the simulation a complex superstructure. The advantages of this procedure are related to the possibility of modelling structures with complex geometries which are not invariant along any direction, as well as the capability of performing nonlinear analyses. The main drawback of these models is their high cost in terms of time and computational resources.

# 2 Brief description of the methodology

This paper presents the implementation and application of a numerical methodology which can be used in the prediction of train induced vibrations. In spite of being presented in the context of problems involving soil-track interaction, it can be adopted in the study of generic three-dimensional problems of dynamic SSI for the case of superstructures lying on the surface of the soil. The presented procedure is formulated in the time domain and enables the consideration of complex geometries and nonlinearities in the simulation of the behaviour of the superstructure.

The presented methodology is based on the division of the system into two independent substructures: a) the superstructure and b) its supporting soil. As referred to above, the superstructure is simulated using the FEM while the soil is modelled by means of the fundamental Green functions of a halfspace or a layered halfspace with linear elastic behaviour. In this work the soil surface is assumed to be plane and horizontal. The soil-superstructure coupling is established at the interaction surfaces by means of equilibrium and compatibility equations.

## 2.1 Description of the superstructure

The superstructure includes all parts of the system which are located above the soil and whose behaviour is simulated by means of the FEM. The dynamic equation of the superstructure can be written as

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}} + \boldsymbol{C}\,\dot{\boldsymbol{u}} + \boldsymbol{K}\,\boldsymbol{u} = \boldsymbol{P} \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass matrix, damping matrix and stiffness matrix of the superstructure, $\boldsymbol{u}$ is the displacement vector and $\boldsymbol{P}$ is the vector of external forces. A dot over a variable denotes differentiation with respect to time.

## 2.2 Description of the soil

The soil behaviour is described by means of the fundamental Green functions, which define the dynamic response of the medium due to an unit load applied at any point. The developed tool enables the possibility of considering the soil as a homogeneous halfspace with linear elastic behaviour.

In this procedure the continuous time histories of the soil pressures are approximated by a sequence of rectangular pulses whose value corresponds to the average of the initial and final values of the respective time step.

The soil-superstructure interaction surface is discretized with interaction elements and a uniform pressure is assumed within each element. This division is defined in accordance with the FE mesh in such a way that each interaction element corresponds to a face of a finite element contacting with soil.

The dynamic response of the surface of the soil due to pressure loads (applied in the surface associated with the interaction elements) with rectangular time-dependence must be calculated. This response is calculated by subtracting the responses due to a pair of loads with Heaviside time-dependence applied at consecutive time steps.

The displacements at the surface of the soil due to a point load with Heaviside time-dependence applied at the surface of the soil can be calculated analytically in time-space domain for the case of a homogeneous halfspace [9]. Nevertheless, these analytical solutions neglect the effect of damping and are limited to some values of the Poisson's ratio.

When dealing with a layered halfspace, the response is computed numerically using the Direct Stiffness Method available in the Elastodynamics Toolbox - EDT [10, 11]. In this case the Green functions are calculated in the frequency-wavenumber domain and are then transformed to time-space domain.

As explained above, this method requires the calculation of the soil response due to a pressure load with Heaviside time-dependence. When the soil response is calculated analytically, the solution due to a pressure is obtained by spatially integrating the solution due to a point load. When the Green functions are calculated in the frequency-wavenumber domain, the desired response can be obtained using a more efficient procedure that consists on the solution by the wavenumber content of the loaded area and by the frequency content of a Heaviside load before the transformation to the time-space domain.

In this manner, the matrix $\boldsymbol{H}^j$ $(\boldsymbol{H}^{j<1} = \boldsymbol{0})$ is used to calculate the soil displacement vector $\boldsymbol{w}^j$ observed at time step $t^j = j\,\Delta t$ due to an interface pressure $\boldsymbol{q}$ applied at the time $t = 0$ and kept constant (Heaviside time dependence), being

$$\boldsymbol{w}^j = \boldsymbol{H}^j\,\boldsymbol{q} \tag{2}$$

Considering the matrix $\boldsymbol{F}^k$ defined as

$$\boldsymbol{F}^k = \frac{\left(\boldsymbol{H}^{k+1} - \boldsymbol{H}^{k-1}\right)}{2} \tag{3}$$

equation (2), which relates the soil displacements at a specific time step with the time evolution of the soil pressures, can be re-written as

$$\boldsymbol{w}^i = \sum_{j=2}^{i}\left(\boldsymbol{F}^{j-1}\,\boldsymbol{q}^{i-j+1}\right) + \boldsymbol{F}^0\,\boldsymbol{q}^i \tag{4}$$

## 2.3 Soil-superstructure coupling

The soil-superstructure coupling is established at the interaction surfaces by enforcing the equilibrium of forces and making the displacements compatible.

It is imposed that the displacements at each interaction point (soil) are compatible with the displacements at the corresponding point of the FE model. The relation between the nodal displacements of the FE, $\boldsymbol{u}$, and the displacements of the interaction points (superstructure), $\boldsymbol{v}$, can be written as

$$\boldsymbol{v} = \boldsymbol{T}_u\,\boldsymbol{u} \tag{5}$$

where $\boldsymbol{T}_u$ represents a transformation matrix whose terms result from the FE shape functions evaluated at the interaction points. Thus, the displacements are made compatible by imposing

$$\boldsymbol{w} = \boldsymbol{v} \tag{6}$$

On the other hand, the equilibrium of forces at the interaction surface is guaranteed by imposing that the action of the soil on the structure has the same magnitude as the action of the structure on the soil and opposite signal. Thus, it is necessary to calculate the nodal interaction forces, $\boldsymbol{Q}$, corresponding to the soil pressures, $\boldsymbol{q}$, by means of the following equation

$$\boldsymbol{Q} = -\,\boldsymbol{T}_q\,\boldsymbol{q} \tag{7}$$

where $\boldsymbol{T}_q$ represents a transformation matrix which results form the integration of the FE shape functions over the area of the interaction elements.

Adding the interaction forces, $\boldsymbol{Q}$, to equation (1) one has

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}}^i + \boldsymbol{C}\,\dot{\boldsymbol{u}}^i + \boldsymbol{K}\,\boldsymbol{u}^i = \boldsymbol{P}^i - \boldsymbol{T}_q\,\boldsymbol{q}^i \tag{8}$$

Taking into account equation (4) and performing some mathematical work, equation (8) can be written as

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}}^i + \boldsymbol{C}\,\dot{\boldsymbol{u}}^i + (\boldsymbol{K} + \boldsymbol{K}^{act})\,\boldsymbol{u}^i = \boldsymbol{P}^i + \boldsymbol{Q}^{hist} \tag{9}$$

where

$$\boldsymbol{K}^{act} = \boldsymbol{T}_q \left(\boldsymbol{F}^0\right)^{-1} \boldsymbol{T}_u \tag{10}$$

and

$$\boldsymbol{Q}^{hist} = \boldsymbol{T}_q \left(\boldsymbol{F}^0\right)^{-1} \sum_{j=2}^{i} \left(\boldsymbol{F}^{j-1}\ \boldsymbol{q}^{i-j+1}\right) \tag{11}$$

The influence of the supporting soil on the behaviour of the system is represented by means of the dynamic stiffness matrix of the soil $\boldsymbol{K}^{act}$ and the vector of the historical interaction forces $\boldsymbol{Q}^{hist}$. Since full contact is assumed (equation (6)), the stiffness matrix $\boldsymbol{K}^{act}$ is calculated and added to the stiffness matrix of the structure only once before the first time step. On the other hand, as the vector $\boldsymbol{Q}^{hist}$ changes with time, it has to be calculated and added to the system of equations before processing each time step. The dynamic equation of the system, defined by (9), can be solved by a classical time step integration procedure as the Newmark Method.

## 2.4 Symmetry of the stiffness matrix of the soil

According to [7], when all the interaction elements are identical and the interaction stresses are assumed to be uniform within each element, the matrix $\boldsymbol{F}^0$ is symmetric. However, these conditions do not guarantee the symmetry of the stiffness matrix of the soil $\boldsymbol{K}^{act}$. Thus, it is possible to conclude that $\boldsymbol{K}^{act}$ is symmetric only when $\boldsymbol{F}^0$ is symmetric and simultaneously the following condition is ensured

$$\boldsymbol{T}_q^T = c\,\boldsymbol{T}_u \tag{12}$$

where $c$ is a scalar. In this work, equation (12) is only valid when the elements involved in the interaction are 8-node solid elements.

## 2.5 Damping

Damping cannot be neglected in the simulation of the behaviour of structures and soil. In the structure the damping is considered viscous (frequency dependent) and defined by means of a Rayleigh damping matrix (which results from a linear combination of mass and stiffness matrices). On the other hand, damping in the soil is assumed as hysteretic (frequency independent). The calculation of Green functions enables the consideration of this damping model by means of the following complex parameters [10, 11]

$$(\lambda + 2\mu)_C = (\lambda + 2\mu)(1 \pm D_P\, i) \tag{13}$$

$$\mu_C = \mu\,(1 \pm D_S\, i) \tag{14}$$

where $\lambda$ and $\mu$ are the Lamé constants and $D_P$ and $D_S$ represent the hysteretic material damping ratio for the dilatational waves and the shear waves, respectively.

### 2.6 Response of points lying on the surface of the soil

It is important to note that, knowing the time history of the interaction stresses, the calculation of the response on the surface soil is very simple. In fact, the displacement of a point $\xi^*$ lying on the surface of the soil at the instant $t^j = j\,\Delta t$ is given by

$$\boldsymbol{w}_{\xi^*}(j\,\Delta t) = \sum_{i=1}^{j} \boldsymbol{F}_{\xi^*}^{j\text{-}i}\,\boldsymbol{q}^i \tag{15}$$

where $\boldsymbol{F}_{\xi^*}^{k}$ is obtained as described previously.

## 3 Implementation aspects

### 3.1 Stability of the process

In the developed tool, the dynamic equation of the system, defined by (9), is solved by means of the Newmark Method. In dynamic problems formulated by the FE method, the Newmark Method is unconditionally stable. However, in the present problem, the coupling of the structure and the supporting soil changes this scenario. The stability of the coupled methodologies (FEM-BEM/GF) has been studied by some authors [7, 12]. In their works, the authors concluded that the relation between the time discretization and the spatial discretization influences the stability of the process. In particular, Bode [7] suggests a reference time step $\Delta t$ equal to

$$\Delta t = \frac{d}{2\,C_R} \tag{16}$$

where $d$ represents the maximum distance between points of each interaction element (for square elements $d$ is the diagonal) and $C_R$ represents the Rayleigh wave velocity of the half-space.

In general, the process is stable when the reference value for time step is used. However, for a time step smaller than the reference one the process does not yield accurate results.

One feature that can help in the stability of the procedure is the consideration of the damping of the soil. In the numerical example presented in the section 4 of this work, the presence of damping in the soil turns the process stable. The stability wouldn't be achieved even if a time step ten times greater than the reference one was used. In the proposed methodology, the damping of the soil is considered in the calculation of the Green's functions. As referred to above, one uses hysteretic damping (i.e., damping which is independent of frequency) by considering complex elastic parameters for the soil [10, 11].

In equation (16), it can be observed that the smaller the time step $\Delta t$ is, the smaller the discretization width $d$ of the interface surface must be. For problems involving moving loads at large speeds, the accurate reproduction of the variation of the main variables (displacements and stresses) requires a small time step $\Delta t$ and therefore a small value of $d$. If the FE mesh is defined with each interaction element in correspondence with one finite element, the total number of degrees of freedom (DOF) in equation (9) becomes very large. This might cause the exhaustion of the available memory of the computer or lead to unacceptably long execution times. For this reason, one allows that each finite element contacts with more than one interaction element, or in other words, the interaction elements mesh and the FE mesh might not match at the interaction surface. With this procedure, the time step $\Delta t$ and the width $d$ of the interface elements can be reduced without increasing the number of DOFs of the FE mesh. However, the number of DOFs associated to the soil and the computational effort of computing the convolutions in equation (11) increase.

### 3.2 Relaxed coupling

In some problems, the dominant degrees of freedom are the vertical ones, i.e., the relevant loads and displacements follow the vertical direction. In this kind of problems, sometimes it is reasonable enough to consider that the structure can slide horizontally in relation to the underlying ground. In such problems, the compatibility of displacements is enforced only in the vertical direction and it is assumed that the structure only transmits vertical pressure to the ground (this case is usually referred as relaxed coupling, whereas when no simplification is done it is named full coupling). Hence, the soil variables (tractions and displacements) are reduced to one third and so the time required to evaluate the convolution represented in equation (11) is also reduced. For the problem presented in the section 4 of this work, the full coupling or relaxed coupling conditions are schematically represented in Figure 1.

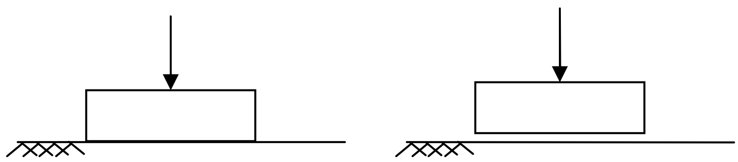


**Figure 1: Full coupling vs. relaxed coupling**

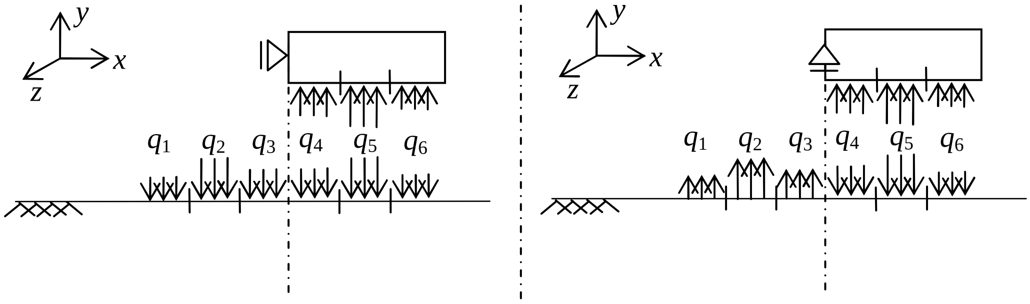
### 3.3 Symmetric structures

A wide range of structural problems involves structures that have symmetric geometries. In these cases, the analysis of the problem can be performed by analyzing two structures that consist in a half part of the original structure. In one of the structures, the displacements perpendicular to the plane of symmetry at the same plane are fixed and the acting load consists on the symmetric part of the load. In the other structure, the displacements in the directions of the plane of symmetry are fixed at the same plane and the acting load consists on the anti-symmetric part of the load. These cases correspond to the symmetric and anti-symmetric cases, respectively. The total response of the structure is obtained by combining the response of the analysis of each structure.

When the problem involves soil-structure interaction, considering the adequate boundary conditions in the soil involves blocking some displacements inside the soil. In principle, the displacements must be fixed within all the depth of the soil (until infinity in the case of a half space), and so, following this approach, the problem becomes more complicated than the original one. However, the initial problem can still be simplified if one only imposes the proper boundary conditions to the structure, while for the ground one imposes symmetry/anti-symmetry of the applied tractions. The needed impositions are represented in Figure 2 both for the symmetric and anti-symmetric case.

Concerning the algorithm of the procedure, this type of analyses only requires that the flexibility matrices be condensed by adding (or subtracting) columns of respective *mirror* elements. As an example and considering only the vertical components of displacements and tractions, it is shown next how to obtain the flexibility matrices for the symmetric case. For the anti-symmetric case, the previous solution is multiplied by -1. The values $f_{ij}$ represent the transfer functions between element *i* and element *j*.

$$\begin{Bmatrix} w_4 \\ w_5 \\ w_6 \end{Bmatrix} = \begin{bmatrix} f_{41} & f_{42} & f_{43} & f_{44} & f_{45} & f_{46} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} & f_{56} \\ f_{61} & f_{62} & f_{63} & f_{64} & f_{65} & f_{66} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{Bmatrix} = \begin{bmatrix} f_{44}+f_{43} & f_{45}+f_{42} & f_{46}+f_{41} \\ f_{54}+f_{53} & f_{55}+f_{52} & f_{56}+f_{51} \\ f_{64}+f_{63} & f_{65}+f_{62} & f_{66}+f_{61} \end{bmatrix} \begin{Bmatrix} q_4 \\ q_5 \\ q_6 \end{Bmatrix}$$




| | | | | | |
|---|---|---|---|---|---|
| $q_{1,x} = -q_{6,x}$ | $q_{2,x} = -q_{5,x}$ | $q_{3,x} = -q_{4,x}$ | $q_{1,x} = q_{6,x}$ | $q_{2,x} = q_{5,x}$ | $q_{3,x} = q_{4,x}$ |
| $q_{1,y} = q_{6,y}$ | $q_{2,y} = q_{5,y}$ | $q_{3,y} = q_{4,y}$ | $q_{1,y} = -q_{6,y}$ | $q_{2,y} = -q_{5,y}$ | $q_{3,y} = -q_{4,y}$ |
| $q_{1,z} = q_{6,z}$ | $q_{2,z} = q_{5,z}$ | $q_{3,z} = q_{4,z}$ | $q_{1,z} = -q_{6,z}$ | $q_{2,z} = -q_{5,z}$ | $q_{3,z} = -q_{4,z}$ |

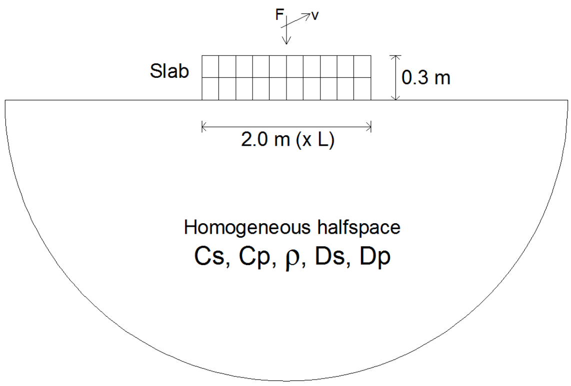
**Figure 2: Traction equivalencies for symmetric (left) and anti-symmetric (right) cases**

Using the symmetry (anti-symmetry) conditions, one can solve two problems with approximately half of the dimension of the original problem, saving time and memory.

## 4 Numerical example

In this section the presented methodology is applied to the study of the dynamic response of a slab resting on a homogeneous halfspace due to the passage of a moving load. The obtained results are compared with those computed by means of a 2.5D FEM/ITM methodology [5, 13].

The geometry and the properties of the slab as well as the properties of the homogeneous halfspace are shown in Figure 3. Different lengths of the model, $L$, were considered in the calculations. The results presented in this paper correspond to a length $L$ equal to 50 m.




Slab

$E = 30\times10^9$ Pa

$\rho = 2145$ Kg/m$^3$

$\nu = 0.2$

Homogeneous halfspace

$C_S = 250.0$ m/s

$C_P = 433.0$ m/s

$\rho = 1800$ Kg/m$^3$

$D_S = D_P = 0.02$

**Figure 3: Geometry and properties of the slab-soil system.**

A vertical load of magnitude $F$ equal to 1 kN moving over the length of the slab, $L$, at different speeds, $v$, is considered in this study. The presented results correspond to a circulating speed equal to 125 m/s, which is below the velocity of the surface waves of the soil.

At a first stage the slab is discretized into two layers of $8\times200$ identical 20-node solid elements with dimensions $0.25\times0.25\times0.15$ cubic meters. The chosen mesh leads to a $8\times200$ regular discretization of the interaction surface. Based on (16), a time step, $\Delta t$, equal to $10^{-3}$ s is adopted. Figure 4 shows the evolution of a point on the top surface of the slab (corresponding to a node of the FE mesh at the mean section of the model) with the load position. This figure shows a comparison between the obtained results with those calculated using a 2.5D FEM/ITM model. A good agreement between the results of both models can be observed. A small disturbance in the curve obtained in the current work can be identified, which occurs when the load enters the model. In a similar representation, Figure 5 shows the evolution of the vertical displacement of two points located 2 m and 5 m off the border of the slab. Once again a good agreement between the results is observed. Nevertheless, the disturbance between the analysed point and the slab increases. This fact leads to the conclusion that the study of the response at larger distances from the slab requires a more extensive model.

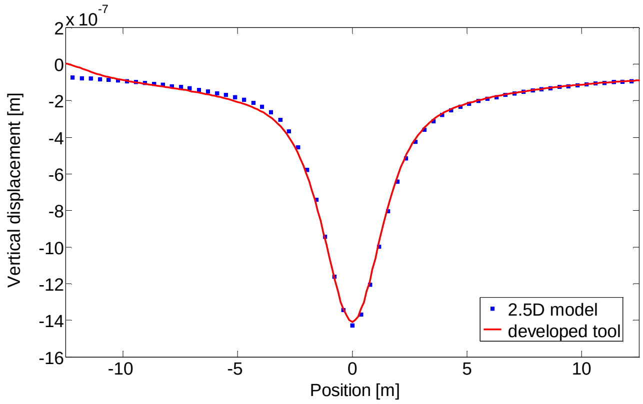



**Figure 4: Vertical displacement of a point of the slab: comparison between the obtained results and those computed using a 2.5D FEM/ITM model.**

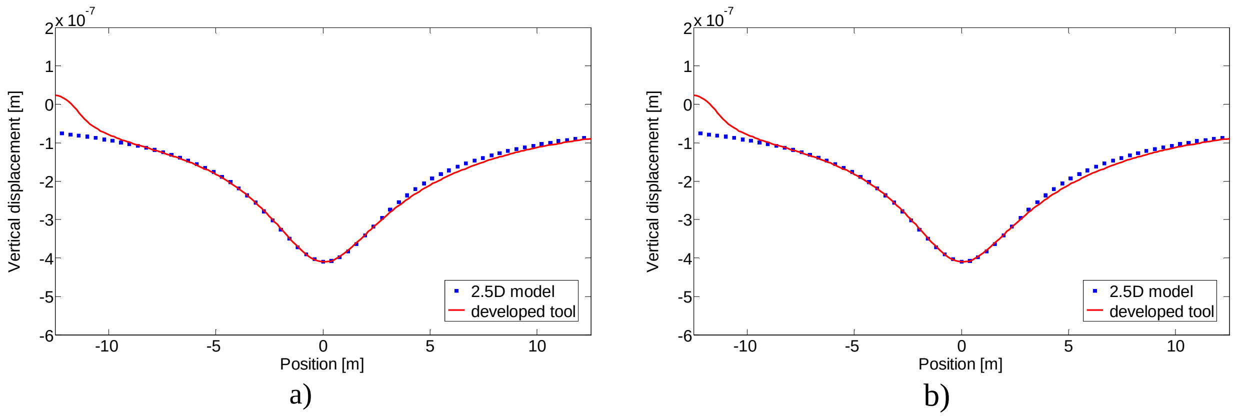
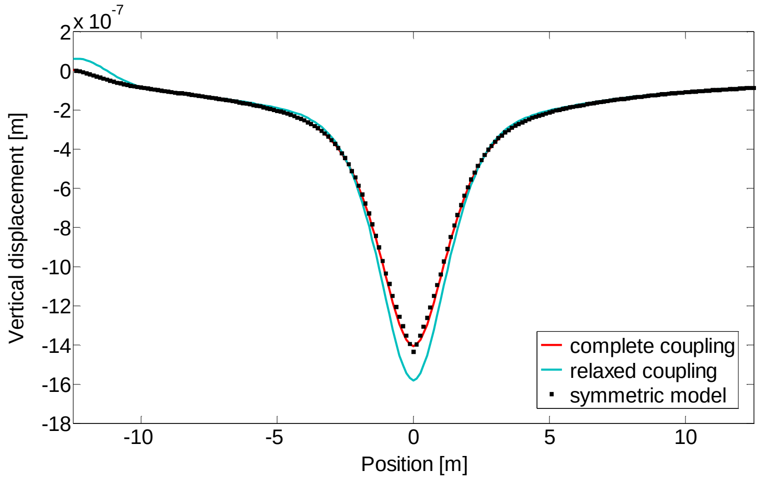


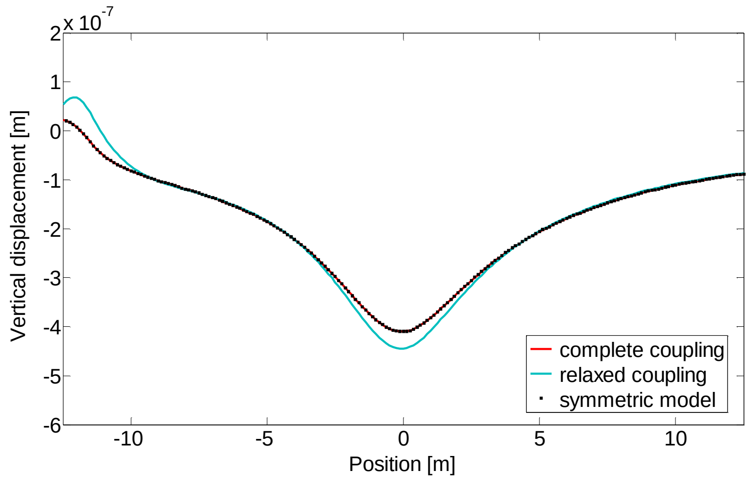

**Figure 5: Vertical displacement of a point a) 2 m and b) 5 m off the border of the slab: comparison between the obtained results and those computed using a 2.5D FEM/ITM model.**

At a second stage, the influence of considering relaxed coupling is considered. In other words, the same problem is studied but only a vertical coupling is assumed. The discretization adopted for the slab (FE mesh) as well as for the interaction surface are equal to those used in the previous case. As a consequence, the same time step is considered.

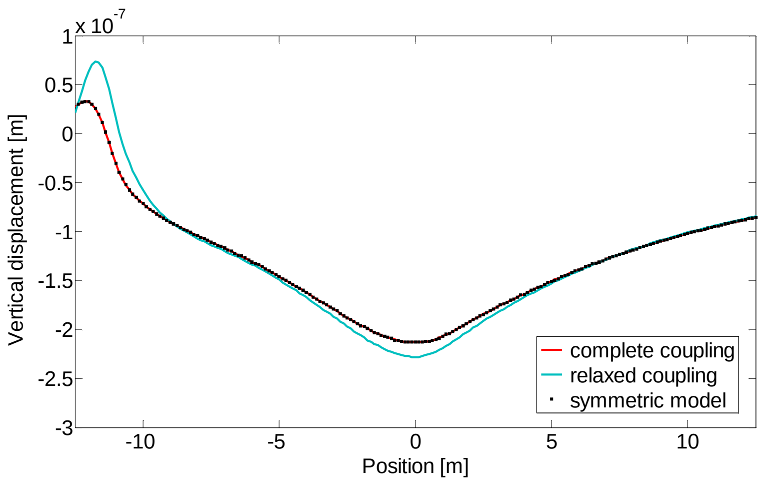
Finally, in order to show a more efficient way to study this problem a third case is presented. In this analysis the symmetry of the superstructure and the loading is considered and consequently only half of the structure is modelled. It is also considered that each finite element in the interface surface contacts with 4 interaction elements, which enables the use of the same time step and larger finite elements. Thus, the slab is discretized into two layers of $4\times100$ identical 20-node solid elements with dimensions $0.50\times0.50\times0.15$ cubic meters. According to the previous considerations, this mesh leads to a $8\times200$ regular discretization of the interaction surface.




a)




b)




c)

**Figure 6: Vertical displacement of a) a point of the slab; b) a point 2 m off the border of the slab; c) a point 5 m off the border of the slab: comparison of the results obtained with three distinct approaches.**

Figure 6 shows a comparison of the results obtained for the three cases. As expected, the results obtained in the first and third analyses are almost coincident. The symmetry assumption as well as the consideration of several interaction elements per finite element (and consequently larger finite elements) lead to a more efficient computation. The comparison between the results obtained considering full coupling and relaxed coupling shows a small difference in terms of the maximum value. As expected the greater value occurs for a relaxed coupling. It is also possible to observe that the referred difference is more evident in the points of the slab (FE mesh) and decreases when the distance between the analysed point and the slab increases.

# 5 Conclusions and further developments

This paper briefly presents the implementation and application of a numerical analysis methodology which can be used in the prediction of train induced vibrations. A numerical example, based on the study of the dynamic response of a slab subjected to the passage of a moving load, is presented. The comparison of the obtained results with those obtained by means of a different validated method reveals a good agreement. The consideration of some simplifications and/or assumptions leads to good results and makes the methodology more efficient from the computational point of view. Since this tool was developed in order to be used in the prediction of vehicle induced vibrations, some tests with increasing complexity are still needed until that goal is achieved.

# Acknowledgements

This paper reports research developed under the financial support of "FCT - Fundação para a Ciência e Tecnologia", Portugal. The first and second authors wish to thank FCT for the financial support provided by the grants SFRH/BD/24197/2005 and SFRH/47724/2008, respectively.

The authors also wish to acknowledge the support of the project "Risk Assessment and Management for High-Speed Rail Systems" of the MIT - Portugal Transportation Systems Area.

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