SECOND-ORDER OPTIMIZATION OF FRAMES WITH NONLINEAR BEHAVIOR

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PROBLEM

- Minimize the cost of a portal frame

FRAME BEHAVIOR

- Liner elastic between plastic hinges

- Plastic hinges with limited plastic rotation

- Plastic hinges are automatically located
  in the most critical positions

OPTIMIZATION APPROACH

- Solve of a nonlinear program
- Second-order approximation
- Integrated formulation
- No sensitivity analysis
- All the problem variables are present in the nonlinear program

OPTIMIZATION SOFTWARE

- NEWTOP
- General purpose code
- Lagrange-Newton method
- Symbolic manipulation of all the functions

NONLINEAR PROGRAMMING

Minimize $f(\underset{\sim}{x})$

subject to

$$\underset{\sim}{g}(\underset{\sim}{x}) \leq \underset{\sim}{0} \qquad \rightarrow \qquad g_i(\underset{\sim}{x}) + s_i^2 = 0$$

$$\underset{\sim}{h}(\underset{\sim}{x}) = \underset{\sim}{0}$$

- Variables / functions ⇨ real and continuous
- All the functions are generalized polynomials, such as:

$$f(\underset{\sim}{x}) = 5.9\,x_1^2 x_4^{-3} - 3.1 x_2 + 2.7\,x_1^{-1} x_3 x_5^2 - 1.8$$

GENERALIZED POLYNOMIALS

$$f(\underset{\sim}{x}) = 5.9\,x_1^2 x_4^{-3} - 3.1x_2 + 2.7\,x_1^{-1} x_3 x_5^2 - 1.8$$

- A symbolic manipulation is performed
- Expression parsing and evaluation is simplified
- Exact first and second derivatives can be easily calculated
- All these operations can be efficiently performed

INPUT FILE

```
### Main title of the nonlinear program
      Symmetric truss with two load cases (kN,cm)
Min.
    +565.685 * t5 ^ 2 + 100 * t8 ^2 ; # truss volume (cm3)

s.t.i.c.
    Min. area 4:   - t4 ^ 2 + 0.15 < 0 ;

s.t.e.c.
    Equil 16:  + 141.421 * t5 ^ 2 * disp16 - 100 = 0 ;

END_OF_FILE
```

- All the software is coded in ANSI C

LAGRANGIAN

$$L\left(\underset{\sim}{X}\right) = f\left(\underset{\sim}{x}\right) + \sum_{k=1}^{m} \lambda_k^g \left[g_k\left(\underset{\sim}{x}\right) + s_k^2 \right] + \sum_{k=1}^{p} \lambda_k^h h_k\left(\underset{\sim}{x}\right)$$

VARIABLES

$$\underset{\sim}{X} = \left(\underset{\sim}{s}, \underset{\sim}{\lambda}^g, \underset{\sim}{x}, \underset{\sim}{\lambda}^h\right)$$

SOLUTION

- Stationary point of the Lagrangian

SYSTEM OF NONLINEAR EQUATIONS

$$\nabla L\left(\underset{\sim}{X}\right) = \underset{\sim}{0} \quad \Rightarrow \quad \left|\begin{array}{ll} 2 s_i \lambda_i^g = 0 & (i = 1, \ldots, m) \\ g_i + s_i^2 = 0 & (i = 1, \ldots, m) \\ \dfrac{\partial f}{\partial x_i} + \sum_{k=1}^{m} \lambda_k^g \dfrac{\partial g_k}{\partial x_i} + \sum_{k=1}^{p} \lambda_k^h \dfrac{\partial h_k}{\partial x_i} = 0 & (i = 1, \ldots, n) \\ h_i = 0 & (i = 1, \ldots, p) \end{array}\right.$$

- The solution of the system is a KKT solution when

$$\boxed{\underset{\sim}{\lambda}^g \geq 0}$$

LAGRANGE-NEWTON METHOD

- The system of nonlinear equations

$$\nabla L(\underset{\sim}{X}) = \underset{\sim}{0}$$

is solved by the Newton method

- In each iteration the following system of linear equations has to be solved

$$\underset{\sim}{H}\left(\underset{\sim}{X}^{q-1}\right)\Delta\underset{\sim}{X}^{q} + \nabla L\left(\underset{\sim}{X}^{q-1}\right) = \underset{\sim}{0}$$

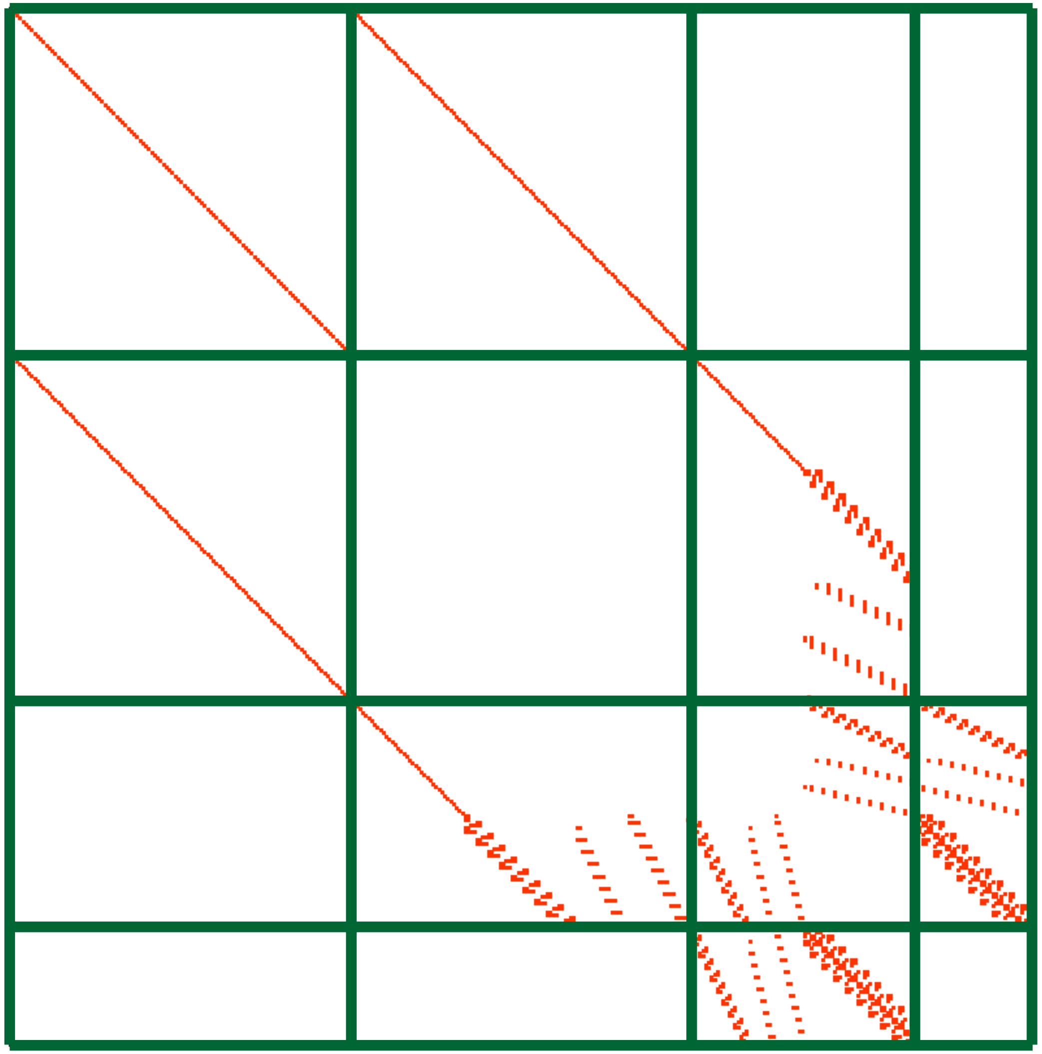
HESSIAN MATRIX

$$\underset{\sim}{H} = \begin{array}{c|cccc} & (m) & (m) & (n) & (p) \\ \hline (m) & Diag\left(2\lambda_i^g\right) & Diag\left(2s_i\right) & \underset{\sim}{0} & \underset{\sim}{0} \\ (m) & & \underset{\sim}{0} & \dfrac{\partial g_i}{\partial x_j} & \underset{\sim}{0} \\ (n) & & & \bullet & \dfrac{\partial h_j}{\partial x_i} \\ (p) & S\,Y\,M\,M\,E\,T\,R\,I\,C & & & \underset{\sim}{0} \end{array}$$

$$\bullet \quad \frac{\partial^2 f}{\partial x_i \partial x_j} + \sum_{k=1}^{m} \lambda_k^g \frac{\partial^2 g_k}{\partial x_i \partial x_j} + \sum_{k=1}^{p} \lambda_k^h \frac{\partial^2 h_k}{\partial x_i \partial x_j}$$

HESSIAN MATRIX SPARSITY PATTERN

$$\underset{\sim}{H} =$$



SYSTEM OF LINEAR EQUATIONS

- Gaussian elimination
  - adapted to the sparsity pattern of the Hessian matrix

- Conjugate gradients
  - diagonal preconditioning
  - adapted to an indefinite Hessian matrix

LINE SEARCH

$$\underset{\sim}{X}^{q} = \underset{\sim}{X}^{q-1} + \alpha \, \Delta \underset{\sim}{X}^{q}$$

NEWTOP COMPUTER CODE

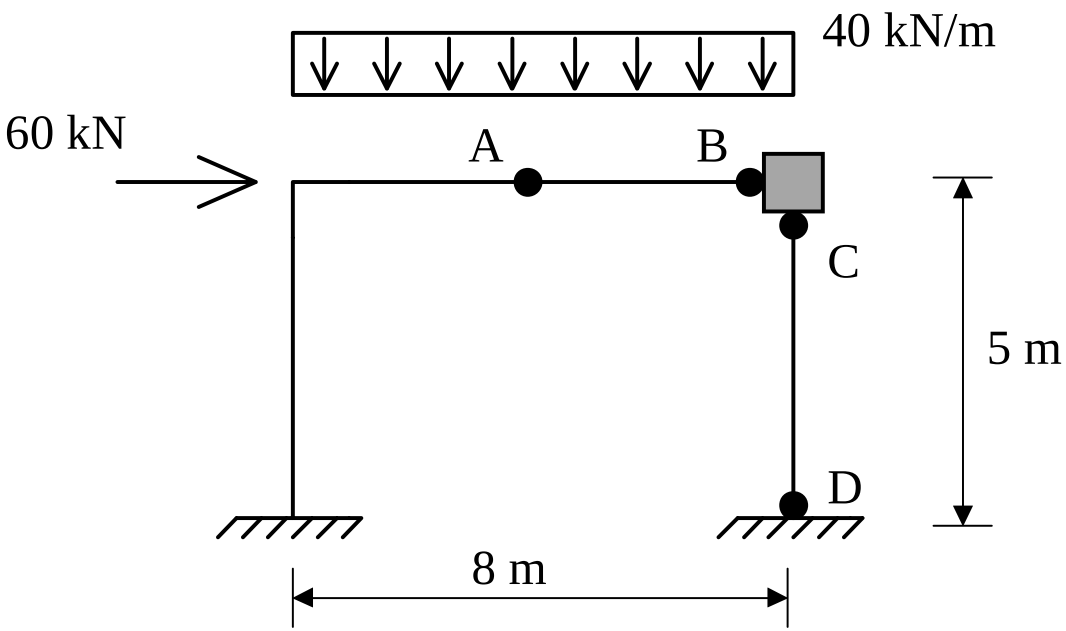
- All the variables are scaled
- Constraints are normalized
- Elementary equality constraints are substituted:

$$x_i = c\,x_j \qquad \text{or} \qquad x_i = c$$

- The NLP is simplified
- Large scale problems can be solved

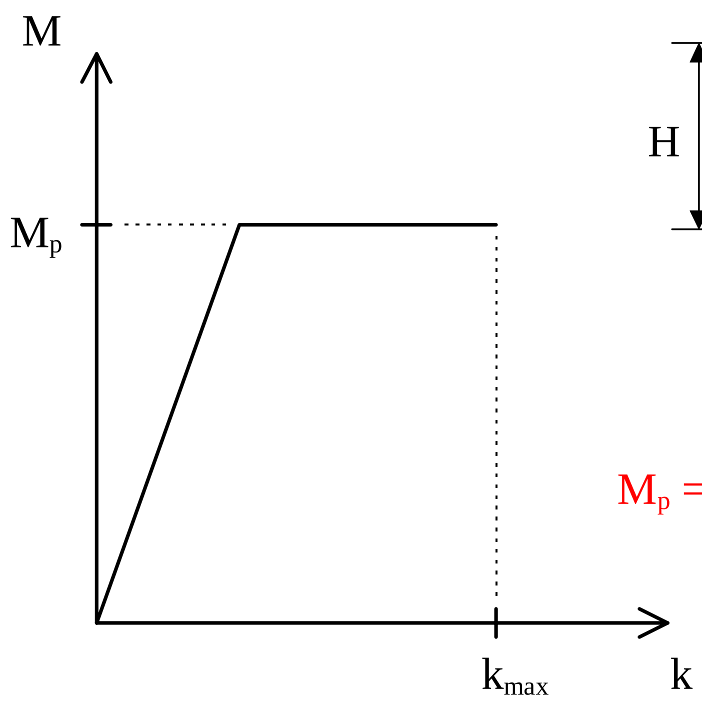
PORTAL FRAME

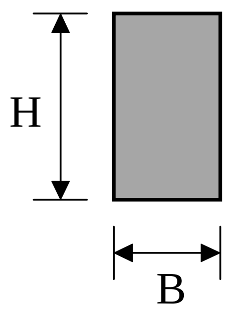
- Cost minimization
- Independent design variables ⇨ cross section parameters



NONLINEAR MATERIAL BEHAVIOR

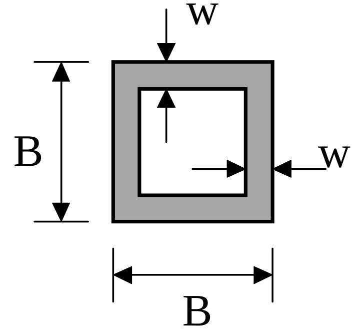
- Linear-perfectly plastic behavior
- Linear-constant moment-curvature diagram





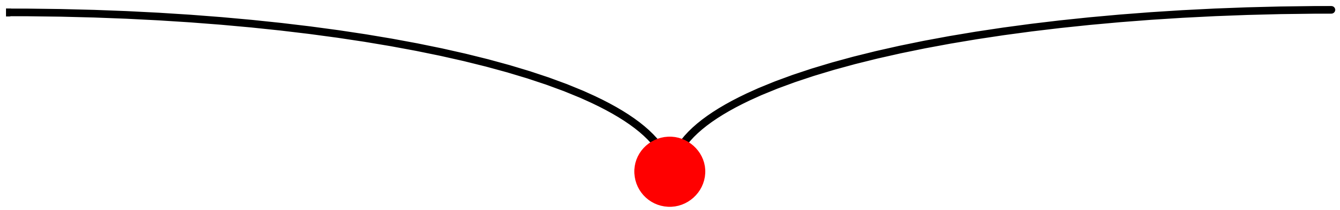
$M_p = \sigma_{max}\, BH^2/4$

$M_p = \sigma_{max}\, w\,(3B^2\text{-}6Bw+4w^2)\,/2$

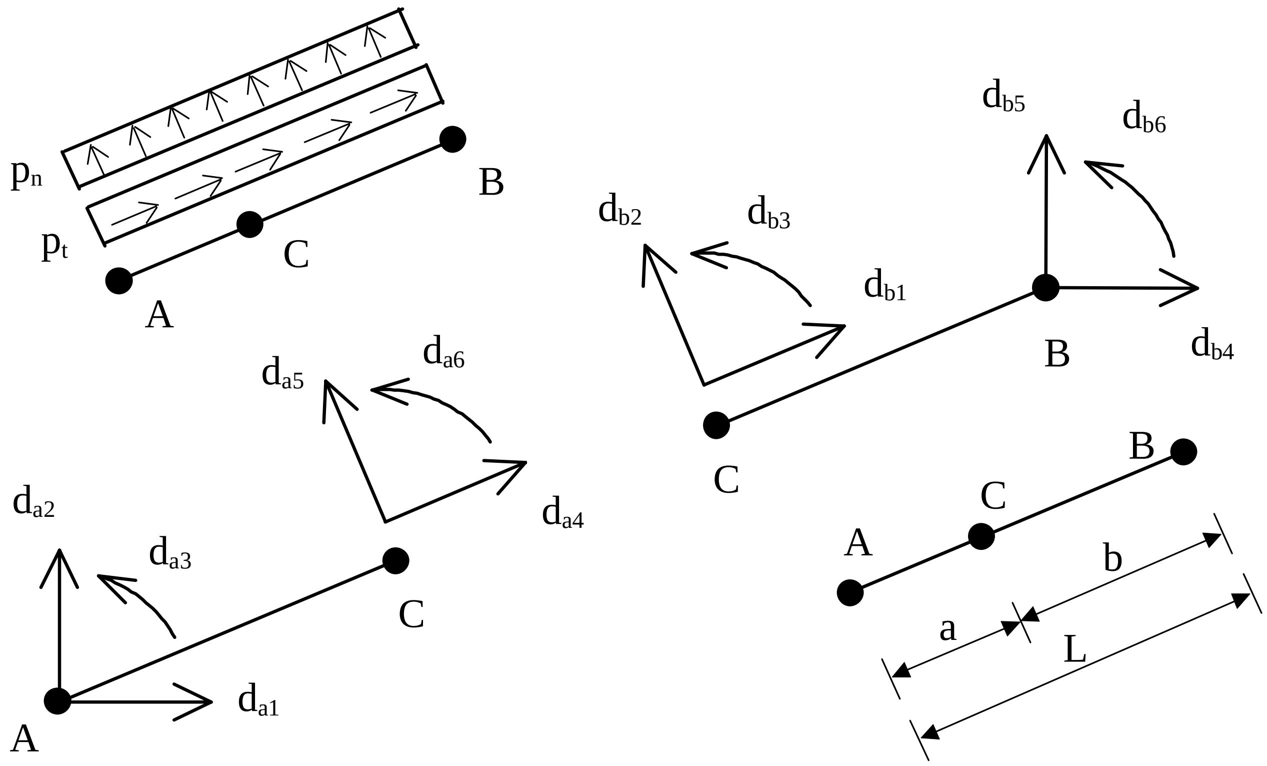


NONLINEAR MATERIAL BEHAVIOR

- Plastic deformations concentrated in plastic hinges
- Plastic hinge rotation may be limited
- Collapse mechanism may not be reached
- Linear behavior between plastic hinges



STRUCTURAL DISCRETIZATION

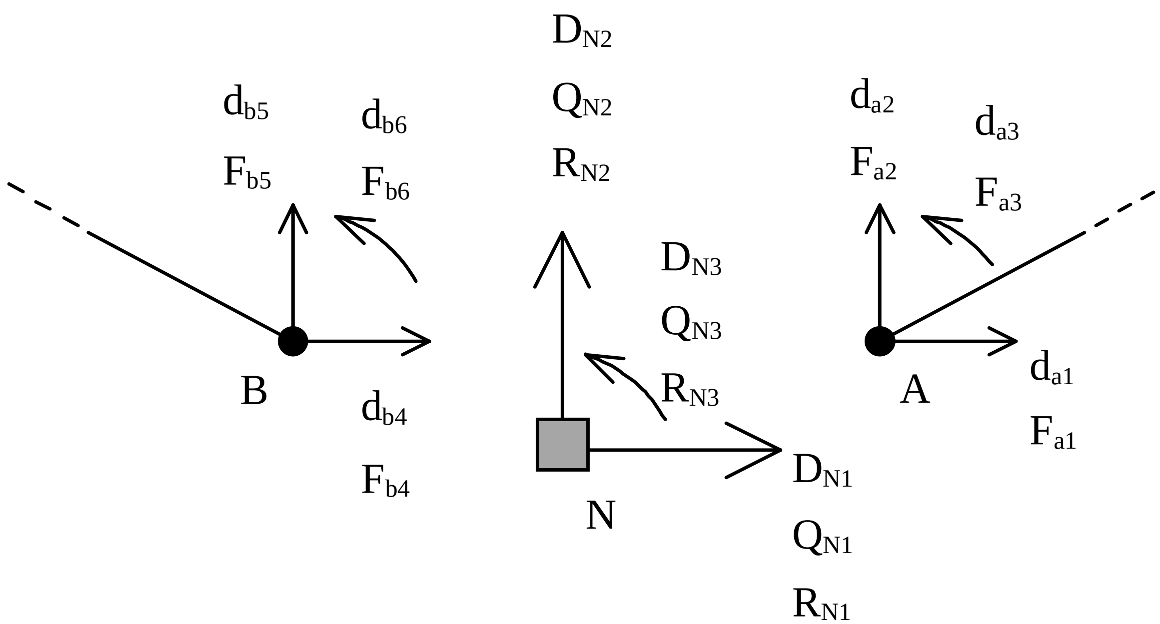


$$\underset{\sim}{K}_a \; \underset{\sim}{d}_a = \underset{\sim}{F}_a + \underset{\sim}{P}_a$$

$$\underset{\sim}{K}_b \; \underset{\sim}{d}_b = \underset{\sim}{F}_b + \underset{\sim}{P}_b$$

- Hinge C ⇨ point of max. bending moment

EQUILIBRIUM EQUATIONS



$$\underset{\sim}{F}_a{}' + \cdots + \underset{\sim}{F}_b{}'' + \cdots = \underset{\sim}{Q} + \underset{\sim}{R}$$

- Reactions are only present in constrained dof 's

NON LINEAR PROGRAM

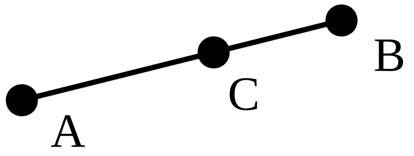
- Objective function: cost ⇨ $f(\underset{\sim}{x}) = \sum_{i=1}^{NB} c_i A_i L_i$

- Equality constraints:
  - beam behavior ⇨ $\underset{\sim}{K}_a \, \underset{\sim}{d}_a = \underset{\sim}{F}_a + \underset{\sim}{P}_a$
  - equilibrium ⇨ $\underset{\sim}{F}_a{}' + \cdots + \underset{\sim}{F}_b{}'' + \cdots = \underset{\sim}{Q} + \underset{\sim}{R}$
  - compatibility ⇨ $d_{a1} = D_{N1}$
  - cross section properties ⇨ $A, I, M_p = \cdots$
  - beam length ⇨ $L = a + b$

- Equality constraints (cont.):
  - plastic hinge rotation ⇨ $\theta_A = d_{a3} - D_{N3}$
  - elastic-plastic complementary in each plastic hinge:

$$\theta_A = 0 \ \text{ or } \ F_{a3} = M_p \ \text{ or } \ F_{a3} = -M_p \ \Rightarrow \ \theta_A\left(M_p^2 - F_{a3}^2\right) = 0$$

  - nodal displacement ⇨ $D_j = \overline{D}$ (only at prescribed dof 's)
  - reaction ⇨ $R_k = 0$ (only at non prescribed dof 's)
  - null shear force in C ⇨ $F_{a5} = 0$ (locates C in the point of max. bending moment)

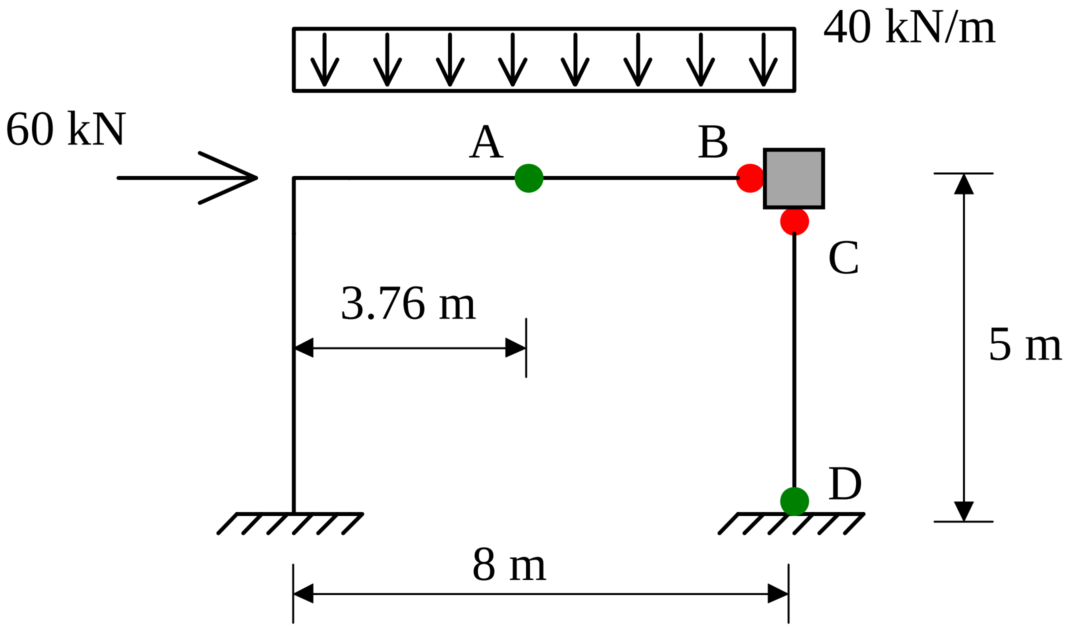


- Inequality constraints:
  - side constraints ⇨ $x_{\min} \le x_i \le x_{\max}$
  - moments are limited by $\pm M_p$ ⇨ $-M_p \le F_{a3} \le M_p$
  - limited plastic hinge rotation ⇨ $-\theta_{\min} \le \theta_C \le \theta_{\max}$

    - limiting values depend on the type of material and on the shape of the cross section

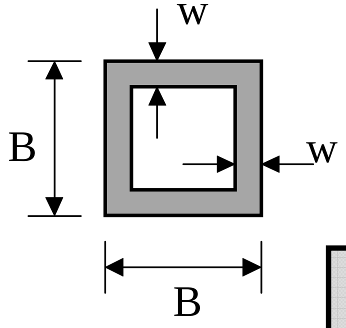
    - crushing, brittle failure and local buckling can thus be avoided

NUMERICAL EXAMPLE



$$E = 200\ GPa$$

$$\sigma_{max} = 250\ MPa$$



$$w = 1\ \text{cm}$$

$$-0.01 \le \theta \le 0.01\ \text{rad}$$

$$B_i \rightarrow \text{independent design variables}$$

NUMERICAL RESULTS

- Optimal solution - linear behavior
  - Volume = 0.175 m^3
  - Horizontal displacement = 2.7 cm
- Optimal solution - nonlinear behavior
  - Volume = 0.157 m^3 (10 % smaller)
  - Horizontal displacement = 5.8 cm (2 x)

CONCLUSIONS

- More realistic approach of the frame design problem
- Ultimate and serviceability conditions may be considered
- More economical structures can be designed
- Friendly user interface is still required
- Solving the nonlinear program is still a *hard* task