Second-order shape optimization of a steel bridge

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Abstract

A comparison is made between the cost minimization of a truss with variable cross-sectional dimensions and fixed geometry (sizing) and the same problem with variable nodal coordinates (shape optimization). Both problems can be formulated as a mathematical program that comprises both structural analysis and design. When this integrated formulation is used, the number of variables in the mathematical program is usually large and an efficient algorithm is required. In this paper the shape optimization problem is solved by a second-order algorithm called NEWTOP that is based on the Lagrange-Newton method. The expressions that define the mathematical program are parsed and their first and second derivatives are symbolically determined and evaluated by the computer program. With the NEWTOP code problems with more than 4 000 independent design variables and 20 000 constraints have already been successfully solved. The solution of a large variety of structural optimization problems has shown that this code is robust, efficient and very accurate. A simple shape optimization example and a bridge design problem are presented. The latter corresponds to a steel pedestrian bridge with eight spans and constant height. Sizing and shape optimization of one of the central spans are performed and compared.

1 Introduction

In the shape optimization of a truss three types of variables can be identified: cross section sizing variables, geometric configuration variables and structural behavior variables. Sizing and configuration variables may be simultaneously treated as design variables (Kirsch[1]) or separately, leading to a multilevel approach (Vanderplaats[2]). In this case two different methods may be applied to the sizing and to the configuration subproblems (Lipson & Gwin[3]). When sizing and geometric variables are grouped and each optimization iteration is decomposed in a sensitivity analysis phase and a design modification phase, the algorithm that calculates the derivatives in order to the design variables must distinguish between both types of variables (Beckers[4]). These derivatives may be calculated analytically or using finite differences (Fleury[5]). More recently an integrated formulation that treats geometric and behavior variables simultaneously in a single mathematical program has been proposed by Burns & Khachaturian[6].

2 Shape optimization of trusses

In this paper an integrated formulation for the cost minimization of planar trusses is presented. All the aforementioned variables are included in a single mathematical program, namely cross-sectional dimensions, coordinates of the points, nodal displacements, etc. (see Section 3.1). Since the displacement method is used to describe the structural behavior, each equilibrium equation is introduced in the mathematical program as an equality constraint. The following inequality constraints have also to be considered: lower bounds on the cross-sectional dimensions, move limits in nodal coordinates, lower and upper bounds on bar stresses and local Euler buckling in each bar. The objective function is the cost of the truss.

The integrated formulation adopted in this work leads to a mathematical program whose size is usually larger than those obtained with other formulations, such as those referred in Section 1. This is a disadvantage in terms of computational burden but leads to a more versatile approach that facilitates the inclusion of new types of constraints and allows for the use of general purpose optimizers. In the next Section the characteristics of the optimizer that was used in this work are briefly described.

3 Optimization with NEWTOP

NEWTOP is a computer code that can be used to calculate the solution of a mathematical program described by an objective function and a set of equality and inequality constraints. These functions are specified in a data file using a human readable format. When the data volume exceeds a few lines of text, the automatic generation of the functions is strongly recommended in order to avoid a time consuming phase where typing errors would certainly occur. A specific computer program was developed whose purpose is the generation of the objective function and constraints that are required to solve the shape optimization problems described below. This program needs the description of the truss topology, materials, cross section types, loads and some optimization related parameters, such as move limits and lower bounds on the cross-sectional dimensions. Since this data is described in a format that is similar to those found in analysis codes, the use of cost minimization techniques by designers that are not familiar with optimization programs is thus facilitated.

The main characteristics of the NEWTOP code are the following (for a more detailed description see Azevedo[7,8]):

- the algorithm is based on the Lagrange-Newton method;
- all the functions are parsed and first and second derivatives are symbolically determined (derivation by finite differences is not required and previously derived functions need not be supplied);
- the Hessian matrix is calculated in each iteration, a Newton direction is used and the corresponding step length is modified by a scalar line search parameter;
- the sparsity pattern of the Hessian matrix is exploited in order to improve the efficiency of the optimization process;
- quadratic convergence is achieved when the current solution is close enough to a stationary point of the Lagrangian;
- inequality constraints are converted to equality constraints by means of the addition of a squared slack variable;
- the Lagrange multipliers of inequality and equality constraints are also calculated during the iteration process;
- scaling is automatically performed;
- elementary equality constraints, such as $x_j = 7.2\, x_i$, are substituted in the nonlinear program before the iteration process;
- the types of functions that are accepted by the computer code are presently restricted to polynomials with positive or negative exponents, such as

$$f\left(x_1,\dots,x_n\right) = 5.9\, x_1^2 x_4^{-3} - 3.1 x_2 + 2.7 x_1^{-1} x_3 x_5^2 - 1.8 \,. \qquad (1)$$

  The code is now being developed in order to accept any type of function.

3.1 Shape optimization variables

The expressions that define the mathematical program may refer variable names instead of the simple representation $x_i \left(i = 1,\dots,n\right)$ shown in eqn (1). These names must be typed as a sequence of characters that are accepted by ASCII text files (e.g., *D_i_k* is the displacement of node *i* in the direction *k*).

Some functions refer the cross-sectional area of the bar (A) and the Euler buckling constraints refer the smallest moment of inertia of each cross section (I). In most cases, A and I can be exactly or approximately calculated with the following expressions

$$A = C_0^A + C_1^A B + C_2^A B^2 \tag{2}$$

$$I = C_0^I + C_1^I B + C_2^I B^2 + C_3^I B^3 + C_4^I B^4 . \tag{3}$$

The value of the C_i^j parameters is fixed a priori and depends on the shape of the cross section. The variable B is an independent design variable that defines the size of the cross section, for example the diameter or the width (see Section 5). The variables A and I are replaced in all the functions of the mathematical program with the expressions on the right-hand side of eqns (2) and (3). It is possible to perform this substitution keeping all the functions in the form of eqn (1).

Figure 1 shows a truss bar, its degrees of freedom and loads.

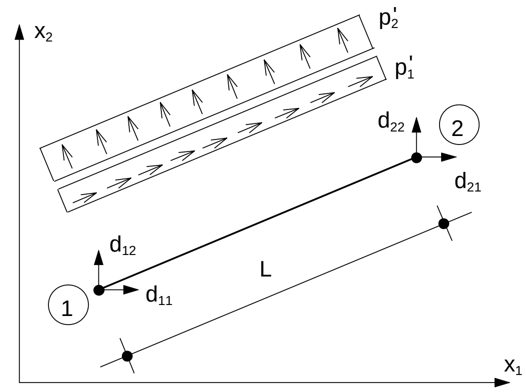

Figure 1. Truss bar and related variables.

Since the bar length is given by

$$L = \sqrt{\left(x_{21} - x_{11}\right)^2 + \left(x_{22} - x_{12}\right)^2} \tag{4}$$

the substitution of this variable in the expressions of the elements of the stiffness matrix (e.g., EAL^{-1}) would generate terms that are not in the form of eqn (1). For this reason the variable L is not substituted and the following equality constraint must be added to the mathematical program

$$- L^2 + x_{11}^2 + x_{12}^2 + x_{21}^2 + x_{22}^2 - 2x_{11}x_{21} - 2x_{12}x_{22} = 0\,. \tag{5}$$

This technique adds N variables and N equality constraints to the mathematical program, being N the number of bars whose length may vary. This drawback will be overcome in the next version of NEWTOP, since the capacity to treat expressions such as eqn (4) will be added.

The displacements at the left and right end of each bar must be equal to the displacements of the corresponding truss node. This relation can be established by the following equality constraints

$$d_{jpk} = D_{ik} \tag{6}$$

where j is the bar number, p is the bar node (1 or 2), i is the corresponding point number and k is the nodal degree of freedom (1 or 2). Variables on the left-hand side of eqn (6) are replaced in the mathematical program before the iteration process. Only the nodal displacements D_{ik} remain as behavior variables.

The integrated formulation presented in this paper treats the following variables during the iteration process:

- B_j - cross-sectional dimension (usually the diameter or the width); j loops over each bar whose cross section may vary;
- x_{iq} - coordinate of a point (i) that can be moved in the direction q;
- L_j - bar length; j loops over each bar whose length may vary;
- D_{ik} - displacement of the node i in the direction k; prescribed degrees of freedom are skipped.

According to the classification referred in Section 1, B_j are cross section sizing variables, x_{iq} and L_j are geometric configuration variables and D_{ik} are structural behavior variables.

3.2 Objective function

The objective function is the cost of the truss, given by

$$Cost = \sum_{j=1}^{n} C_j L_j A_j \tag{7}$$

being n the total number of bars, C_j the cost per unit volume of the bar material, L_j the bar length and A_j the cross-sectional area. In eqn (7) C_j is known a priori, L_j is a variable and A_j is replaced with eqn (2).

3.3 Equality constraints

The nonlinear program contains the following equality constraints:

- for each bar whose length may vary, one equation such as eqn (5);
- for each non-prescribed nodal degree of freedom, one equilibrium equation.

These functions only depend on the variables B_j, x_{iq}, L_j and D_{ik} and can be written in the standard form exemplified by eqn (1).

3.4 Inequality constraints

The following inequality constraints are included in the nonlinear program:

- for each bar whose cross section may vary, one minimum width constraint;
- for each bar, lower and upper bounds on the normal stresses and a Euler buckling constraint;
- for each non-constant nodal coordinate, lower and upper bounds on the coordinate value (move limits).

In the Euler buckling constraint the smallest moment of inertia of the cross section is replaced with the right-hand side of eqn (3).

All the inequality constraints depend only on the variables B_j, x_{iq}, L_j and D_{ik} and can be written in the standard form exemplified by eqn (1).

When a move limit constraint is active at the optimal solution the iteration process must be restarted using the last pseudo-optimal solution as a starting point. In some situations the move limits have to be adapted to the characteristics of the problem.

4 Two bar truss

A classical shape optimization problem taken from the literature was used to validate the proposed approach. This problem consists of a symmetric two-bar truss, whose details are described in Figure 2 (see also Svanberg[9]).

This problem has one sizing variable (A) and one geometric variable (x). The dead load is neglected and there is one load case consisting of a single force (see Figure 2). The allowable stress is 100 kN/cm^2. The objective function to be minimized is the truss volume. Lower and upper bounds on A or x do not become active at the optimal solution. At the starting point A = 1.5 cm^2 and x = 50 cm.

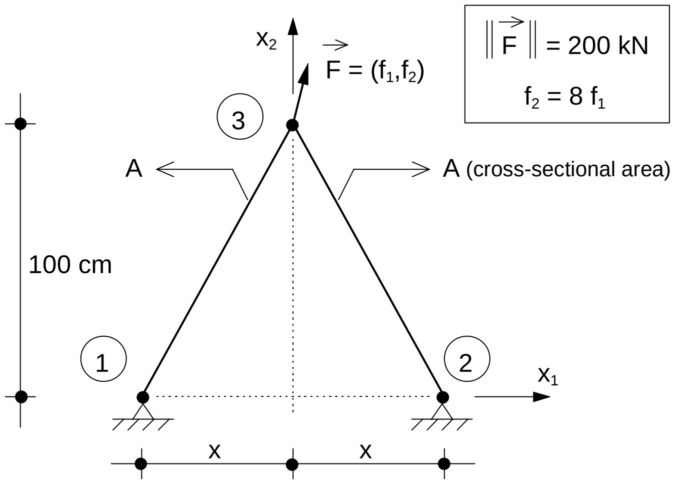

Figure 2. Two bar truss.

The optimal solution was found in 6 Newton iterations that required 0.1 seconds in a RISC workstation. Since the convergence rate is quadratic, a very accurate solution was obtained. The minimum volume of the truss is reached when A = 1.412 027 cm^2 and x = 37.707 243 cm. Svanberg[9] indicates a solution with A = 1.41 cm^2 and x = 38 cm.

5 Steel bridge

The high cost of civil engineering structures offers a good motivation to dedicate some time during the design phase searching a solution that is acceptable and economical. When the same design is reused, the investment in the application of optimization techniques becomes even more effective. In order to evaluate the savings that can be obtained with the shape optimization of a structural design, a steel bridge is optimized under two distinct assumptions. In the first situation the configuration is fixed and only the cross-sectional sizes are allowed to change (see Section 5.1). The second approach consists in the simultaneous optimization of the configuration and cross-sectional dimensions (see Section 5.2). The characteristics that are shared by both approaches are briefly described.

The motivation for this study was the preliminary design of a pedestrian steel bridge to be built in Portugal. The bridge has six spans with 36.0 meters and two extreme spans with 25.0 meters, all having similar characteristics (see Figure 3). A central span with null rotations at its endings and an extreme span with similar boundary conditions were optimized. Only the study of the central span is presented here.

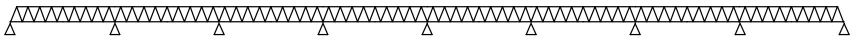

Figure 3. Steel bridge with 8 spans.

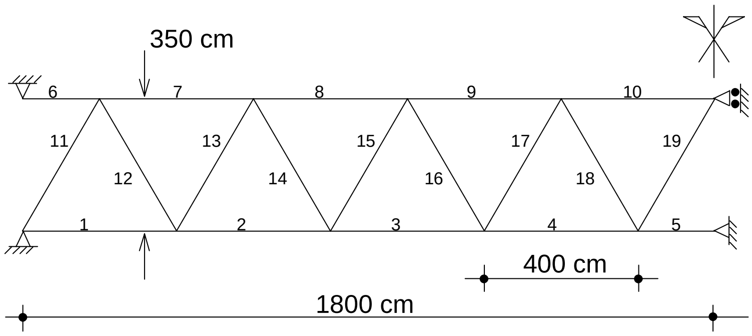

Figure 4. Steel bridge: one half of a central span.

Figure 4 shows one half of a central span of the steel bridge. On the right, symmetry conditions are introduced. On the left, no vertical displacement and no global rotation is allowed. The cross section of all the bars is square and hollow, as shown in Figure 5.

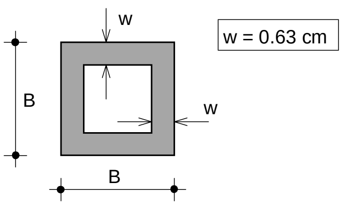

Figure 5. Steel bridge: cross section of the bars.

Web thickness (w) is the same for all bars and its value is 0.63 cm. All the horizontal bars share the same cross-sectional area (group I) and all the diagonal bars are also linked (group II). According to the Portuguese structural codes and to the adopted type of steel, the value of the Young modulus is $E = 20\,600$ kN/cm^2 and the allowable stress is $f_{yd} = 27.5$ kN/cm^2 (tension and compression). In the Euler buckling constraint an additional safety coefficient whose value is 1.8 has to be introduced. The weight per unit volume is the mean value multiplied by 1.35, i.e., $\gamma = 1.0395 \times 10^{-4}$ kN/cm^3. Bars 1 to 5 are subject to a vertical distributed load whose value is $p = 0.11025$ kN/cm. In order to simulate the behavior of the whole structure, the load in bar number 5 had to be concentrated in its left end node. Otherwise half of its resultant would go directly to the support at the right end of the bar. The value of the concentrated load is $p \times L_5$.

5.1 Sizing

The structure shown in Figure 4 was initially optimized with fixed nodal coordinates. The independent design variables are the cross-sectional sizes (B) of the bars belonging to groups I and II. At the optimal solution B_1 = 10.13328 cm, B_{11} = 9.73978 cm and the volume is 169501 cm^3. Only two constraints are active (Euler buckling in bars 1 and 11).

5.2 Shape optimization

In the alternative approach all the non-supported nodes were allowed to move horizontally. The sizing variables B_1 and B_{11} representing groups I and II were also allowed to change. The optimal shape is shown in Figure 6.

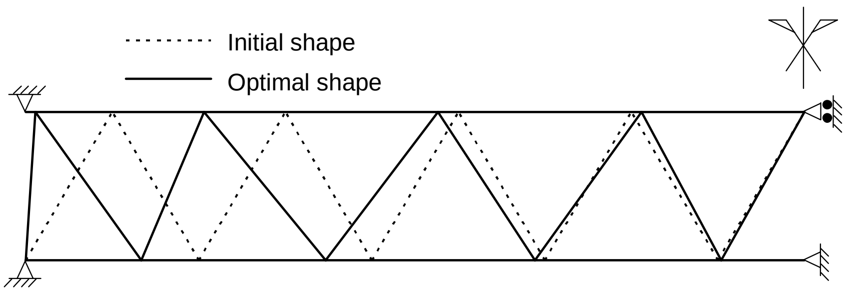

Figure 6. Steel bridge: optimal configuration.

At the optimal solution B_1 = 8.55497 cm, B_{11} = 8.62598 cm and the volume is 146050 cm^3. Only Euler buckling constraints are active (bars 1, 2, 9, 10, 11, 13 and 15). The use of shape optimization techniques allowed for a 14% reduction in the truss cost, relatively to the optimal solution with fixed nodal coordinates. The complexity of the optimal shape did not increase and does not imply additional costs associated with assembly difficulties. Using a RISC workstation or a modern personal computer, the solution shown in Figure 6 can be obtained in less than 10 seconds.

Conclusions

This example has shown that shape optimization can be used as a valuable tool in the search for a structural solution that is both acceptable and more economical than traditional designs. The accuracy and the efficiency of the second-order algorithm used in this work is promising and indicates that this code may be applied to larger and more complex shape optimization problems. The simplicity of the formulation allows for the utilization of the proposed tool in everyday engineering practice.

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