Development of an efficient finite element model for the dynamic analysis of the train-bridge interaction

S. Neves, A. Azevedo & R. Calçada
Faculty of Engineering, University of Porto, Porto, Portugal

ABSTRACT: The design of high-speed railway bridges comprises a set of demands, from safety and serviceability aspects, to new types of equipment and construction solutions. In order to perform an accurate and realistic evaluation of the corresponding dynamic behavior, adequate analysis tools that take into account the complexity of the train-bridge system are required. These computational tools must be based on efficient algorithms to allow for the completion of detailed dynamic analyses in a reasonable amount of time. The classical methods of analysis may be unsatisfactory in the evaluation of the dynamic effects of the train-bridge system and fully assessment of the structural safety, track safety and passenger comfort. A direct and versatile technique for the simulation of the train-bridge interaction was implemented in the FEMIX code, which is a general purpose finite element computer program. The presented case study is an application of the proposed formulation, which proved to be very accurate and efficient.

1 INTRODUCTION

The dynamic behavior of railway bridges carrying high-speed trains can be analyzed with or without the consideration of the vehicle's own structure. The simulation of the train-bridge system requires several independent submeshes and the consideration of contact conditions that represent their interaction.

The formulation of the contact between nodal points of the vehicle and internal points of a finite element is briefly described in this paper. Dynamic equilibrium equations in non prescribed degrees of freedom, in contact degrees of freedom and in prescribed degrees of freedom are separately developed. Contact compatibility equations between points of the vehicle and internal points of a finite element are also separately developed. All these equations constitute a single system of linear equations involving displacements, contact forces and reactions as unknowns. After the solution of this system of linear equations the displacements, velocities and accelerations at the current time step can be calculated and a new time step is started. This heterogeneous system of linear equations can be efficiently solved by means of the consideration of several submatrices with specific characteristics. These techniques have been implemented in FEMIX 4.0, which is a general purpose finite element computer program (Azevedo et al, 2003).

The new formulation is applied to the analysis of the dynamic behavior of the São Lourenço bridge, which is a bowstring arch bridge. The bridge is located in the North Line of the Portuguese railway system, in a section that was recently upgraded to allow the passage at greater speeds of the Alfa pendular train. The numerical model was validated with the results of the ambient vibration test and those measured during several passages of the Alfa pendular train. The dynamic response of the train-bridge system is calculated for train speeds exceeding the allowable speed for that section of the North Line. The corresponding results provide precise indications on the structural safety, track safety and passenger comfort.

2 HHT METHOD WITH TRAIN-BRIDGE INTERACTION

A simple example is used to introduce the types of degrees of freedom that are considered in the formulation of the vehicle-structure interaction in the context of a time step of the Hilber-Hughes-Taylor method (see Fig. 1). On the right, a simply supported beam with two spans ($B1$ and $B2$) is subjected to the contact of a vehicle, shown on the left. The structure of the vehicle is also composed of two beams ($B3$ and $B4$). Nodes 7, 8 and 9 are internal points of the beam $B1$. The location of these nodes may change between time steps, depending on the position of the vehicle. Eventual gaps between both structures (g_i) can be easily considered in the compatibility equations, as will be shown later.

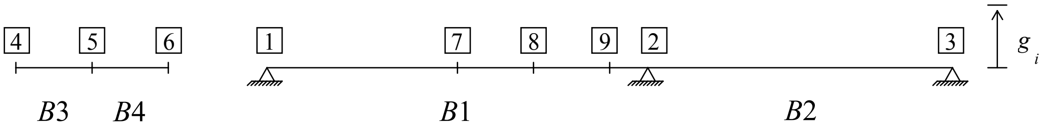


Figure 1. Vehicle and structure: beams and nodal points.

In each nodal point two degrees of freedom are considered (vertical displacement and rotation). Figure 2 shows the generalized displacements in nodal points (1 to 12), the generalized displacements of the contact points of the structure (13, 14 and 15), the interaction forces in the vehicle (X_7, X_9 and X_{11}) and the interaction forces in the structure (Y_{13}, Y_{14} and Y_{15}). The interaction only involves the translational degrees of freedom.

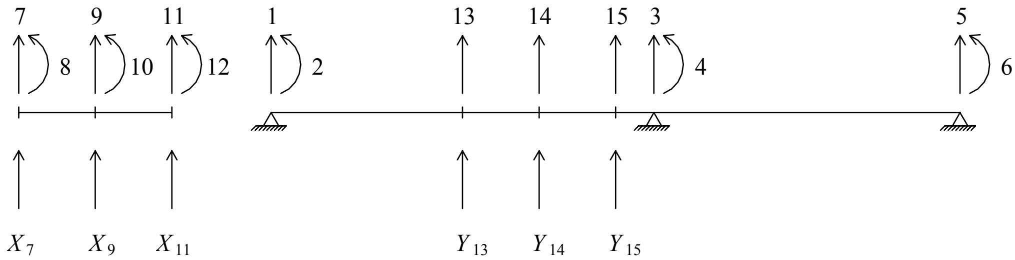


Figure 2. Vehicle and structure: degrees of freedom and interactions forces.

The following classification of the degrees of freedom is considered:

- F – free;
- X – interaction (vehicle);
- P – prescribed;
- Y – interaction (structure).

This classification is used later in this section.

In the context of the Hilber-Hughes-Taylor method (HHT), the dynamic equilibrium equation that involves the degrees of freedom in nodal points (1 to 12) is the following

$$\underline{\mathbf{M}}\,\underline{\ddot{\mathbf{u}}}^{c} + (1+\alpha)\,\underline{\mathbf{C}}\,\underline{\dot{\mathbf{u}}}^{c} - \alpha\,\underline{\mathbf{C}}\,\underline{\dot{\mathbf{u}}}^{p} + (1+\alpha)\,\underline{\mathbf{K}}\,\underline{\mathbf{u}}^{c} - \alpha\,\underline{\mathbf{K}}\,\underline{\mathbf{u}}^{p} = (1+\alpha)\,\underline{\mathbf{F}}^{c} - \alpha\underline{\mathbf{F}}^{p} \qquad (1)$$

In this equation $\underline{\mathbf{M}}$ is the mass matrix, $\underline{\mathbf{C}}$ is the damping matrix, $\underline{\mathbf{K}}$ is the stiffness matrix, $\underline{\mathbf{F}}$ are the applied generalized forces, $\underline{\mathbf{u}}$ are the generalized displacements and α is the main parameter of the HHT method. When $\alpha = 0$ the HHT method reduces to the Newmark method, and for other values of the parameter α, numerical energy dissipation is introduced in the higher modes. The superscript c indicates the current time step ($t + \Delta t$) and the superscript p indicates the previous one (t).

According to Figure 2 and to the classification indicated above, the F type degrees of freedom are the following: 2, 4, 6, 8, 10 and 12. The X type degrees of freedom correspond to the "supports" of the separated vehicle structure, being the following: 7, 9 and 11. The P type degrees of freedom are the main structural supports 1, 3 and 5. The Y type degrees of freedom 13, 14 and 15 consist on the internal displacements of beam $B1$ at the contact points.

According to Azevedo et al (2007), the dynamic response can be obtained by solving the following system of linear equations

$$\begin{bmatrix} \overline{\mathbf{K}}_{FF} & \overline{\mathbf{K}}_{FX} & (1+\alpha)\underline{\mathbf{d}}_{FX} \\ \overline{\mathbf{K}}_{XF} & \overline{\mathbf{K}}_{XX} & -(1+\alpha)\underline{\mathbf{I}}_{XX} \\ (1+\alpha)\underline{\mathbf{c}}_{XF} & -(1+\alpha)\underline{\mathbf{I}}_{XX} & -(1+\alpha)\underline{\mathbf{f}}_{XX} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{u}}_F^c \\ \underline{\mathbf{u}}_X^c \\ \underline{\mathbf{X}}_X^c \end{bmatrix} = \begin{bmatrix} \overline{\underline{\mathbf{F}}}_F \\ \overline{\underline{\mathbf{F}}}_X \\ \underline{\mathbf{g}}_X \end{bmatrix} \quad (2)$$

It is possible to demonstrate that in Equation (2) the coefficient matrix of the system of linear equations is symmetric.

3 APPLICATION TO THE DYNAMIC ANALYSIS OF A BOWSTRING ARCH BRIDGE

3.1 *Dynamic model of the bridge*

The São Lourenço bridge is located at the km +158.662 of the North Line of the Portuguese railway system, in a recently upgraded section for the passage of the Alfa pendular train at a maximum speed of 220 km/h.

The bridge is a bowstring arch composed of two half-decks spanning 42 m with a single track in each (REFER, 2003). Figure 3 shows a view of the bridge. Each deck consists of a 40 cm thick prestressed concrete slab suspended by two lateral arches (see Fig. 4). The arches are linked in the upper part by transversal girders with a rectangular hollow section and diagonals in double angles that assure the bracing of the arches.

Figure 3. São Lourenço bridge and Alfa pendular train.

The bridge was modeled with 2D beam elements. The influence of the deck-track composite effect in the dynamic behavior of railway bridges, due to the transmission of shear stresses between the deck and the rails at the ballasted layer, was treated by Ribeiro et al (2007) and considered in the current finite element model (see Fig. 4).

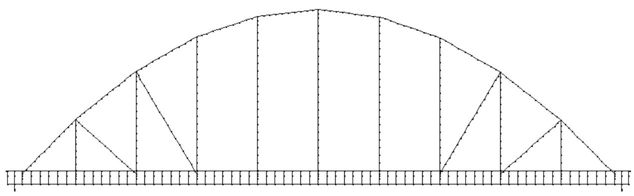
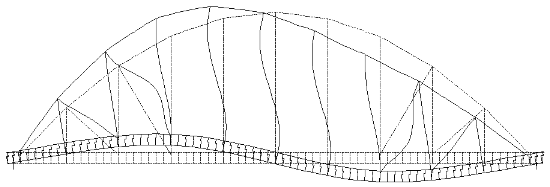


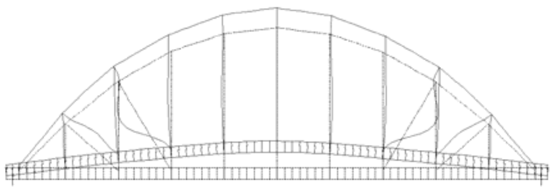
Figure 4. FEM model of the São Lourenço bridge.

The first three vertical mode shapes of the bridge (1V, 2V and 3V) are represented in Figure 5.

1st vertical mode (1V): f = 4.25 Hz



2nd vertical mode (2V): f = 6.20 Hz



3rd vertical mode (3V): f = 9.94 Hz

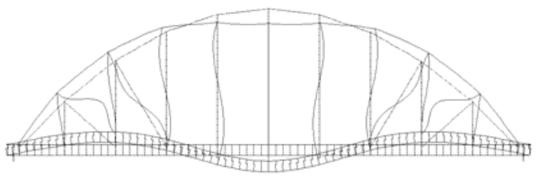


Figure 5. Mode shapes and corresponding frequencies.

3.2 *Dynamic model of the Alfa pendular train*

The Alfa pendular train is composed of six carriages. Each carriage has two bogies with two axles each. The total weight of the composition is 298.3 t, tare weight, and 323.3 t for a regular loading. The maximum weight per axle is 14.4 t and the total length of the composition is 158.90 m.

The dynamic model of the train is shown in Figure 6 and consists of rigid bodies simulating the vehicle box (mass M_c) and the bogies (mass M_b), springs with stiffness K_p (or K_s) and dashpots with damping c_p (or c_s) simulating the primary (or secondary) suspensions, springs with stiffness K_h simulating the wheel-rail contact and masses M_e simulating the axles and wheels. The train is modeled with 2D beam elements (see Fig. 6).

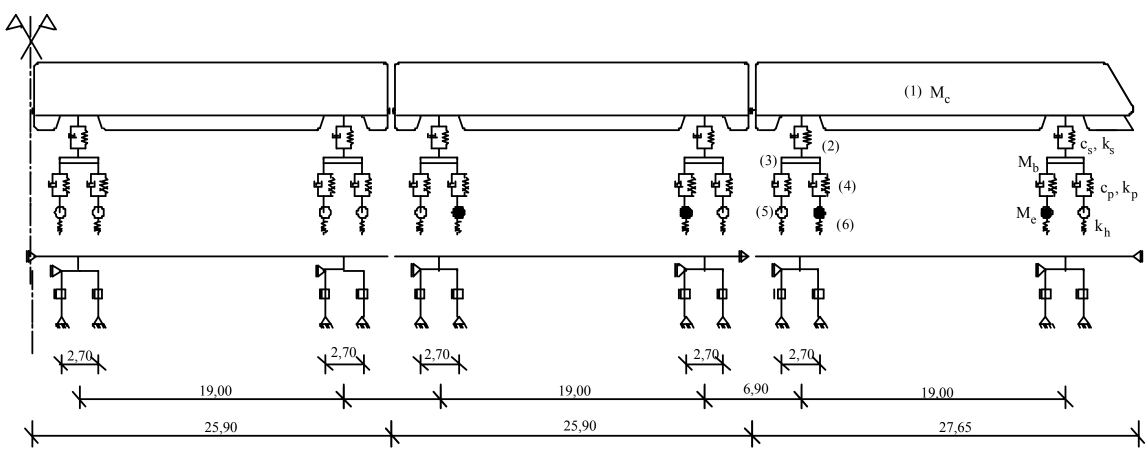


Figure 6. Dynamic model of the Alfa pendular train.

3.3 *Experimental validation of the numerical model*

In order to validate the numerical model an experimental campaign was undertaken, which included an ambient vibration test and several measurements of passages of the Alfa pendular train (Ribeiro et al, 2007). The ambient vibration test was used to identify the dynamic properties of the structure, namely its natural frequencies, mode shapes and damping coefficients. The vertical accelerations at some points of the deck were measured during several passages of the Alfa pendular train. A good agreement between the natural frequencies obtained with the numerical model and the experimental values could be observed (Azevedo et al, 2008).

3.4 *Dynamic response of the train-bridge system*

Several dynamic analyses were performed for speeds ranging between 140 an 420 km/h, with a 5 km/h step, consisting of 57 simulated passages. The dynamic analyses were performed using the HHT method with constant average acceleration, i.e., with $\alpha = 0$, $\beta = 1/4$ and $\gamma = 1/2$ (Hughes, 2000), and a time step equal to 0.002 s. Rayleigh damping is adopted in the present study. The damping ratios of modes 1V and 3V were obtained in the experimental campaign, and are equal to 1.4% and 2.4%, respectively.

Figure 7 shows the maximum vertical acceleration at the span quarter-point and mid-point for the passage of the Alfa pendular train at speeds ranging between 140 and 420 km/h.

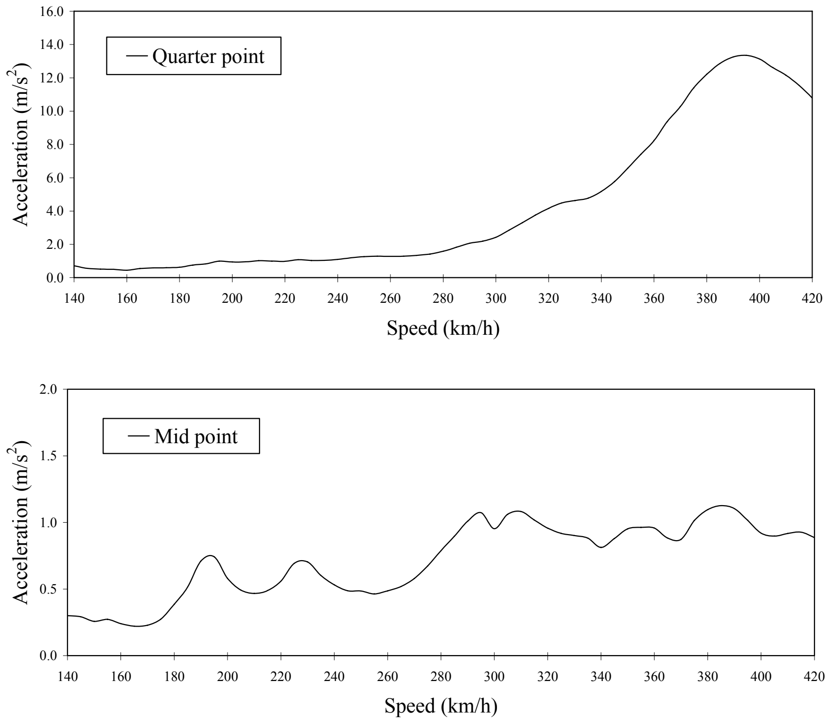


Figure 7. Maximum vertical acceleration at quarter-point and mid-point.

Resonance peaks can be observed in Figure 7 for train speeds of 395 km/h, at the span quarter-point, and for train speeds of 195 km/h, 230 km/h, 295 km/h and 310 km/h, at mid-point.

The resonance phenomena may occur for trains with regularly spaced axles, traveling at critical speeds defined in EN1991-2 (2003).

$$v_{res}(i,j) = \frac{d\, n_j}{i}, \; i = 1,2,3,4,...,1/2,1/3,1/4,... \quad (3)$$

where d is the characteristic length and n_j is the jth natural frequency of the bridge. The characteristic length is a distance (many times repeated) between the vehicles or axles that at some critical speeds may induce a periodic excitation of the bridge and cause the resonance phenomenon.

The characteristic length of the Alfa pendular train is 25.9 m (see Fig. 6). According to Figure 7 and to Equation (3), the critical speeds are: i) taking into consideration the span quarter-point and an excitation of the structure with a frequency which is equal to the mode 1V frequency (v_{res} = 25.9×4.25/1×3.6 = 396 km/h ≈ 395 km/h); ii) considering the mid-point and an excitation with a frequency which is equal to 1/2 and 1/3 of the frequency of mode 2V (v_{res} = 25.9×6.20/2×3.6 = 289 km/h ≈ 295 km/h and v_{res} = 25.9×6.20/3×3.6 = 193 km/h ≈ 195 km/h) and 1/3 and 1/4 of the frequency of mode 3V (v_{res} =25.9×9.94/3×3.6 = 309 km/h ≈ 310 km/h and v_{res} = 25.9×9.94/4×3.6 = 232 km/h ≈ 230 km/h).

The consideration of the train-bridge interaction in the dynamic analysis is particularly useful since it allows for an accurate evaluation of the vibrations of the rail cars, and, consequently, for an assessment of the riding comfort of the passengers. Figure 8 shows the time histories of the vertical acceleration in the first and last car bodies at a speed of 395 km/h, for which resonance of mode 1V occurs.

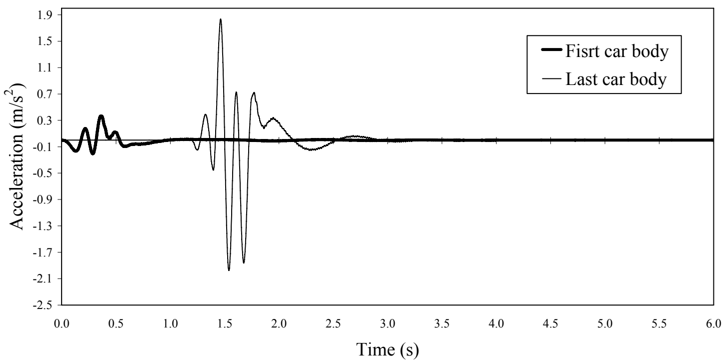


Figure 8. Vertical acceleration in the first car body and last car body.

Figure 9 shows the maximum vertical acceleration in the first and last car bodies of the train for speeds ranging between 140 and 420 km/h. The maximum level of acceleration occurs in the last car body for speeds that correspond to the resonance of the bridge.

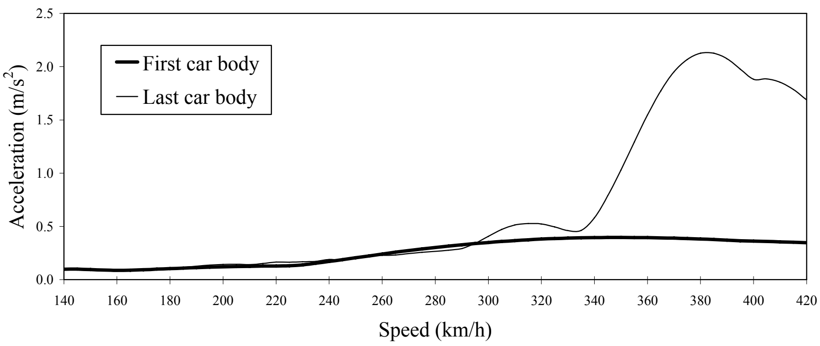


Figure 9. Maximum vertical acceleration in the first car body and last car body.

The dynamic analyses with train-bridge interaction were performed for 57 different speeds, which corresponded to a total execution time of 2624 s (≈44 min) in a 2.13 GHz personal computer. The bridge submesh has 1860 non prescribed degrees of freedom and the train submesh has 192. The total execution time of each analysis corresponds to the period between the entry of the first axle in the bridge and the exit of the last axle.

4 ASSESSMENT OF THE DYNAMIC BEHAVIOR

Based on the regulations of EN1991-2 (2003) and EN1990-A2 (2005), the dynamic response of the bridge is evaluated in terms of structural safety, track safety and passenger comfort. The corresponding results are presented in this section.

4.1 *Structural safety*

The calculation of the dynamic amplification factors, φ'_{dyn}, resulting from the passage of the train at high speeds, is based on the results of the time history analysis of the train-bridge system. These values of φ'_{dyn} are corrected by a factor φ'' in order to take into account for the track irregularities. Table 1 shows a comparison between the maximum vertical displacements, at the span quarter-point and at mid-point, obtained with a dynamic analysis with the Alfa pendular train, and by means of the application of the load model LM 71, being these results multiplied by the corresponding dynamic amplification factor Φ_2.

Table 1. Maximum vertical displacements at the span quarter-point and mid-point.

Model	Quarter-point	Mid-point
	mm	mm
LM 71	12.75	8.01
Alfa pendular	18.95	2.15

4.2 *Track safety*

The norm EN1990-A2 (2005) specifies deflection and vibration limits that must be taken into account in the design of railway bridges. These limits guarantee traffic safety in terms of: i) vertical acceleration of the deck; ii) deck torsion; iii) vertical deformation of the deck; and, iv) horizontal deformation of the deck.

When, for example, the vertical acceleration of the deck exceeds its limit value, instability of the ballast may occur, or even a loss of contact between wheel and rail. For the case of ballasted deck bridges, the peak value of the vertical acceleration of the deck should not exceed 3.5 m/s^2 ($\approx$ 0.35g).

The acceleration limit is violated only at the span quarter-point and for speeds exceeding 310 km/h (see Fig. 7).

4.3 *Passenger comfort*

Since the passenger comfort depends on the vertical acceleration of the car body, the norm EN1990-A2 (2005) limits its value to 1.0, 1.3 or 2.0 m/s^2, which correspond to a comfort level classification of "very good", "good" or "acceptable".

By observing Figure 9 it can be readily concluded that the comfort level at the first car body is "very good" for any speed included in the studied range. For the case of the last car body, the comfort level is "very good" for speeds up to 345 km/h. For speeds near 395 km/h (resonance speed) the comfort level remains close to acceptable.

5 CONCLUSIONS

In the present work an integrated model whose aim is the dynamic analysis of structures by the Hilber Hughes Taylor method is proposed. This algorithm treats the interaction between moving vehicles and a structure such as a bridge. This work provides a significant improvement relatively to the method proposed by Delgado & Cruz (1997) and Calçada (1995), since the compatibility between vehicle and structure is no longer imposed by an iterative method, but by means of an integrated formulation that considers as variables displacements and contact forces. The governing system of equations comprises dynamic equilibrium equations and compatibility equations. The system of linear equations that arises at each time step is efficiently solved by Gaussian elimination, by considering several submatrices and their own characteristics.

The proposed formulation is applied to the dynamic behavior of a bowstring arch bridge. The dynamic response of the train-bridge system was calculated for train speeds only occurring in high-speed lines, allowing for a better understanding of the behavior of the system and its resonance effects, in terms of structural safety, track safety and passenger comfort. A significant improvement in terms of efficiency could also be observed, without compromising the accuracy of the results.

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